

# Supplementary Material

## Defaults and Attention: The Drop Out Effect

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### 1 Example AVF

This example involves two actions, choose the option on the left ( $a_L$ ) or choose the option on the right ( $a_R$ ), and two prizes, a good one ( $x_1$ ) and a bad one ( $x_2$ ), with normalized expected “prize” utility of  $U(x_1) = 1$  and  $U(x_2) = 0$ . The DM knows that one and only one of the two actions will yield the preferred prize  $x_1$ . Which action yields  $x_1$  is determined by state  $\omega \in \{\omega_L, \omega_R\}$ . In state  $\omega_L$  the preferred prize is on the left, while in state  $\omega_R$  it is on the right. We assume that the good prize is ex ante more likely to be on the right, with,

$$\Pr(\omega_R) = \frac{2}{3}.$$

A fully-informed DM who knows the true state of the world would always pick a utility maximizing option. Errors are made only because DMs do not perfectly perceive the state of the world, so they do not fully understand the consequences of the available actions. Before making an action choice, the DM can get up to two signals about the state of the world. Each signal either suggests that the good prize is on the left ( $\sigma_L$ ) or on the right ( $\sigma_R$ ). When the good prize is actually on the left, the signal is correct with probability  $\frac{3}{4}$ , and when the good prize is actually on the right, the signal is also correct with probability  $\frac{3}{4}$ , so that,

$$\Pr(\sigma_L|\omega_L) = \Pr(\sigma_R|\omega_R) = \frac{3}{4}.$$

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Choice of attentional effort depends on a production function that turns this effort into a probability distribution over the number of signals received. We assume that there are logistic returns to attention: signals are received with probability  $\frac{1}{1+e^{-(6\alpha-6)}}$ , and if signals are received, there is an equal probability of getting 1 or 2 signals. This reflects initial increasing returns to attention and then decreasing returns.

To compute optimal attention, we first determine the expected prize utility for different signal realizations by calculating posterior beliefs (after receiving any signals and before making an action choice) about the location of the good prize for all available actions. In the example, when no signals have been received, the right action  $a_R$  is more likely to yield the good prize and is therefore the best choice. The corresponding expected prize utility is,

$$V_0 = \frac{2}{3}.$$

The best choice with one signal depends on its direction. If it indicates that the good prize is on the right (a signal of  $\sigma_R$ ), this confirms  $a_R$  as the better choice. If it indicates that the good prize is on the left ( $\sigma_L$ ), then  $a_L$  becomes the better choice since the signal is more informative than the prior. Overall, the good prize is chosen if and only if the state is  $\omega_L$  and the signal is  $\sigma_L$  or the state is  $\omega_R$  and the signal is  $\sigma_R$ , so the corresponding expected prize utility is,

$$V_1 = \frac{2}{3} \cdot \frac{3}{4} + \frac{1}{3} \cdot \frac{3}{4} = \frac{3}{4}.$$

The outcome with two signals is clear. If they are both  $\sigma_R$  or one is  $\sigma_R$  and the other is  $\sigma_L$ , then  $a_R$  is chosen. If they are both  $\sigma_L$ , then  $a_L$  is chosen. Overall, the expected prize utility with two signals is,

$$V_2 = \frac{2}{3} \cdot \left(1 - \frac{1}{16}\right) + \frac{1}{3} \cdot \frac{9}{16} = \frac{39}{48}.$$

Combining the signal production function with the results above yields the AVF,

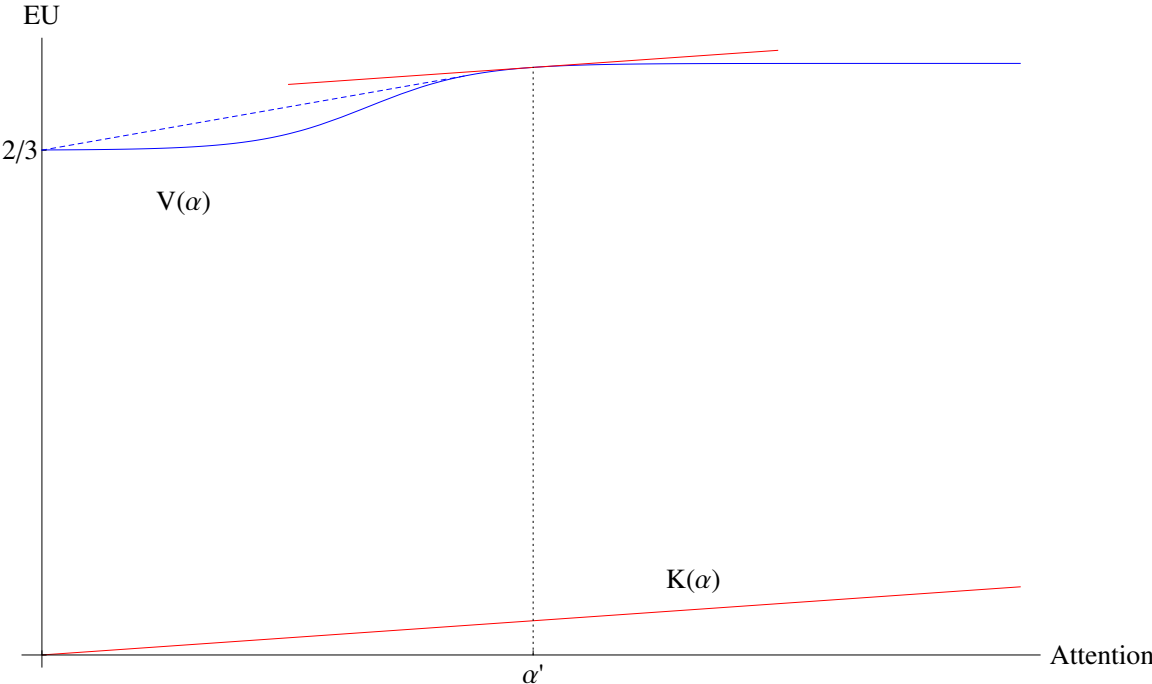
$$V(\alpha) = \frac{1}{1 + e^{-(6\alpha-6)}} \cdot \frac{25}{32}.$$

This AVF is presented in figure 1 below. Note that the AVF resembles a logistic function: it starts out convex and then becomes concave.

## 2 Illustrating the Model

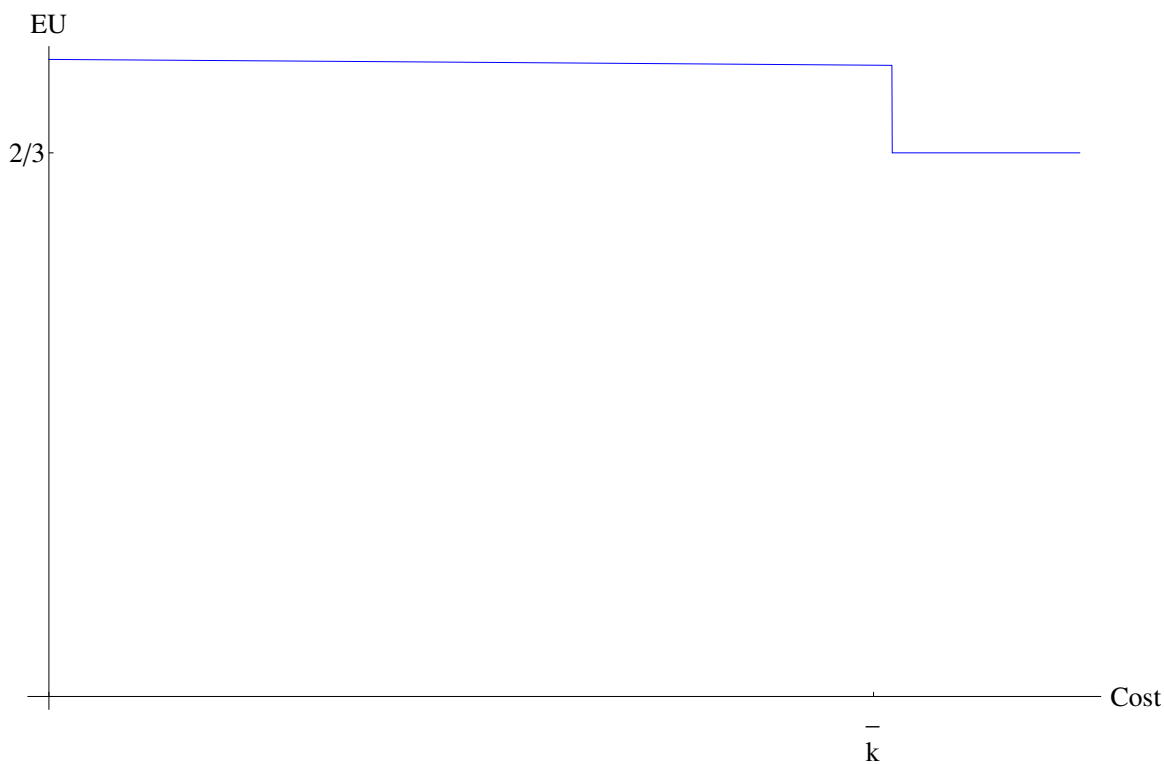
The simple calculus of optimal attention is illustrated in figure 1. For this attentional value function (AVF), the only possible optima are to put in zero attentional effort or to put in an effort level that creates a tangency (at the effort level  $\alpha'$  in the figure) with marginal cost on the concave boundary of the convex hull of the AVF (shown with a dashed line in the figure). To work out which of these is superior, one compares the vertical difference  $V(0) - K(0)$  to  $V(\alpha') - K(\alpha')$ .

Figure 1. Example of the costs and benefits of attention.



In figure 2, we plot  $V(\alpha^*(k))$  for the AVF in figure 1. Note that as the cost of effort rises above zero, there is a gradual reduction in decision quality as the DM chooses lower levels of attentional effort. At a critical cost level, marked as  $\bar{k}$  in the figure, it is optimal to drop out altogether and to choose zero attentional effort.

Figure 2. Choice quality given effort cost.



In figure 3 we show how choice quality can depend on the marginal cost of attention with the high quality default and with the random default. Note that the critical level of attentional cost at which it is optimal to drop out is strictly lower when there is a high quality default than when the default is random, so that  $\bar{k}^R > \bar{k}^H$ .

Figure 3. Choice quality given effort cost.

