Framing as Information Design*

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Abstract

We propose an information design framework for studying the welfare impact of frames. In our model, the frame determines a decision maker's information structure, which summarizes his or her beliefs about the consequences of taking each action. We imagine a policy maker who would like to select the welfare optimal frame based only on the decision maker's choices under each frame. We extend results from the information design literature to show when and how the policy maker can recover both the welfare order over options and the relationship between frames and information structures. We then demonstrate how this allows the policy maker to rank frames according to their informativeness and expected utility. Finally, we consider the welfare impact of information costs by showing how the policy maker can rank frames net of information costs.

Key words: Framing effects, bounded rationality, incomplete information, information design, limited attention

1 Introduction

There is growing interest in the impact of information design (i.e., the information made available to agents) on welfare. This information design literature spans papers on persuasive communication (Kamenica and Gentzkow 2011), the private information of agents in games (Bergemann and Morris

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2016), and inattention to some aspect of the decision problem (for instance, Woodford 2014, Matjka and McKay 2015, Caplin and Martin 2015, Caplin and Dean 2015). In practice, a leading way for policy makers to influence the informativeness of decision makers is by choosing a particular frame (choice architecture) for a decision problem. For instance, they can provide only a short time to make a decision, push some information to a supplementary appendix, or present options in a certain order. In this paper, we use the information design perspective to inform the policy maker's choice of frame. In other words, we imagine the policy maker (framer or nudger) as an information designer.

This joining of literatures is natural for several reasons. For one, as in the nudging literature, the information designer does not want to force a particular choice, just guide the agent into making an otherwise unrestricted choice that best coincides with the designer's objective. As in the framing literature, this is accomplished by changing the agent's information, not the utility of the options themselves. As stated by Bergemann and Morris (2017), "she [the information designer] can achieve this objective even though she has no ability to change the game itself, or force the players to choose particular actions."

As in the framing effects literature, we assume that the objective of the framer is to maximize the welfare of agents, but that the framer does not know the decision makers welfare-maximizing choice, so must infer it from behavioral data. This presents two concerns for policy makers. First, how can choice data be used to identify the welfare optimal option for individuals given that frames may distort choice? Second, how can choice data be used to determine the optimal way to frame options?

In response to these essential questions, several papers have asked whether a stable, frame-independent welfare relation can be inferred from choices despite the distortions produced by frames (Salant and Rubinstein 2008, Bernheim and Rangel 2009, Rubinstein and Salant 2011).[1] When such a relation exists, Benkert and Netzer (2016) show that it can be used also to rank frames.

However, these papers leave several important questions unanswered that our approach can address. For instance, how well informed are decision makers when they make decisions under a particular frame? In the existing literature, it is common to assume that one frame makes it more or less difficult to understand choice options than another frame. Instead, our approach gives the

[1] This is related to a literature on performing welfare assessments in the presence of behavioral biases (Manzini and Mariotti 2007, Apesteguia and Ballester 2015).

conditions on data that allow us to reach a conclusion. In order words, we show when a frame is "revealed" to make options more or less difficult to understand.

We accomplish this by using choice data to reveal the posterior beliefs of agents, which then allows us to identify the information structure of a decision maker for each frame. By looking at how these posterior beliefs change with the frame, it is possible to pinpoint the exact way in which a frame impacts information and hence beliefs. As a consequence, we can rank frames according to the information they induce, which is new to the literature on framing effects. We say that one frame is more informative than another if the revealed information structure with the first frame is more Blackwell informative than the revealed information structure with the second frame.

In addition, policy makers may wonder if choice data can be used to determine the optimal frame inclusive of information costs. They may be concerned that they have potentially harmed decision makers by imposing high information acquisition costs as a result of choosing a particular frame. Using our approach, we can rank frames net of their information costs, which is also new to the literature on framing effects.

There are two additional advantages of our approach relative to existing approaches to framing effects. First, we allow for stochastic choice within frames.[2] Stochastic choice is particularly useful in the context of framing effects because important effects can have subtle effects, such as swinging the probability of making a choice from 10% to 30%.[3] This feature of our approach complements a growing literature that considers stochastic choice to be an essential data set for studying information and utility (such as Manzini and Mariotti 2014, Apesteguia and Ballester forthcoming).

Second, this approach allows our theory to nest all choice distortions that are based on updating beliefs correctly given some unobservable private information. This includes stochastic and partial models of limited attention, such as rational inattention (Sims 2003) and the drift diffusion model (Ratcliff 1978, Krajbich, Oud, and Fehr 2014), which are incompatible with existing approaches to framing effects. Expanding the set of information distortions is likely to be important if we consider settings where information search is "fuzzy." This is particularly relevant given that many applications of nudging and framing take place in settings where choice options are very complex,

[2] Stochastic choice within frames is also modeled in a recent working paper by Bhattacharya, Mukherjee, and Sonal (2017).

[3] This is particularly true when taking the perspective that stochastic choice arises from the choices of many individuals, as many framing effects are measured by the change in the fraction of individuals making each choice.

such as health care plans and retirement plans.

Like Salant and Rubinstein (2008) and Bernheim and Rangel (2009), we make one small, but important, change to the primitives of a decision problem: we add an arbitrary set of frames F.[4] This set can accommodate a wide range of framing effects, such as "the point in time at which a choice is made, the manner in which information or alternatives are presented, the labeling of a particular option as the 'status quo,' the salience of a default option, or exposure to an anchor." (Bernheim and Rangel 2009, p. 55).

However, unlike these papers, our model of framing effects uses information structures to model incomplete information about the available options.[5] We define a "framed information representation", which is when choices can be explained with this model of framing effects. Framed information representations are characterized by simple restrictions on "framed" state-dependent stochastic choice data. Like the deterministic restrictions of Salant and Rubinstein (2008) and Bernheim and Rangel (2009), these restrictions require evaluating choices across frames, and if these restrictions are satisfied, we can posit that the decision maker has frame-dependent information structures and a frame-independent utility function. Armed with these information structures and utility functions, the policy maker is prepared to address some fundamental questions about the optimality of frames.

The rest of the paper is organized as follows. Section 2 places our paper among related papers. Section 3 provides our framework and defines the enriched choice data set that we consider. Section 4 characterizes all data sets that can be rationalized by a frame-independent utility function. When such a utility function exists, section 5 shows how to use the data to rank frames according to their expected utility and their informativeness. Section 6 provides additional conditions on the data consistent with costly information acquisition, in which the frame may impact such costs, and introduces the behaviorally derived ranking of utilities net of these costs. Section 7 concludes.

[4]For Salant and Rubinstein (2008), this is an "extended choice problem", and for Bernheim and Rangel (2009), this is a "generalized choice situation".

[5]Existing papers account for incomplete information in a different way: by subsuming the information of agents into the choice objects themselves (as in Bernheim and Rangel 2009) or by explicitly constructing "consideration sets" using the frame (as in Rubinstein and Salant 2011).

2 Related Literature

Through the main body of the paper, we develop an approach that nests theories of framing effects in which (1) frames do not change the utility of prizes, (2) frames themselves do not communicate decision relevant information, and (3) individuals make optimal choices given their (correct) posterior beliefs based on (potentially stochastic) information about the available options.[6] The first two assumptions are standard in the framing effects literature, and the last assumption is standard in the information design literature (see Bergemann and Morris 2017).

Because of these maintained assumptions, our paper is most closely related to the work of Salant and Rubinstein (2008), Bernheim and Rangel (2009), Rubinstein and Salant (2011), and Benkert and Netzer (2016), who all study how to make inferences about preferences and welfare from framed decision problems. The primary innovation in our paper is to set framing in the context of an information-theoretic decision problem. This approach allows us to use information structures to summarize an agent's beliefs about the set of available options. These structures make it possible to study the informational impact of frames and to nest all information distortions that are based on updating beliefs correctly given some unobservable private information.

Our paper is also closely related to Caplin and Martin (2015), as we extend the decision problem introduced in that paper to a setting with frames. Because Caplin and Martin (2015) do not consider frames, all of the applications to framing effects are new to this paper. In particular, the ranking of frames is entirely distinct. Our paper is also closely related to Caplin and Dean (2015), as it draws from their approach to costly information. However, they treat the utility function as known and do not consider frames, so our application of their methods to the welfare assessment of frames is new and distinct.

This paper also differs from Caplin and Martin (2017), which investigates how attention can vary within a frame due to changes in option quality. Neither the experiment nor the theory presented in Caplin and Martin (2017) include a cross-frame comparison, which is necessary to identify framing effects. As a result, their paper cannot address either framing effects or the informational impact of frames. Also, Caplin and Martin (2017) model a very specialized type of costly attention, so that paper lacks a general theory of costly information.

[6] Because of the second assumption, it follows that frames are independent of prior beliefs about the payoffs to each action, and because of the third assumption, it follows that agents have Bayesian constraints on their posterior beliefs.

3 Framework

3.1 Decision Problems

We add frames to a standard information design decision problem. Following Caplin and Martin (2015) (hereafter CM), we consider not just prizes, but also actions, states, and a probability distribution (prior) over states. Prior probabilities over states are central objects in the information design literature, as they determine the degree to which information structures can distort choices.

Technically, our framework involves a finite set of possible states Ω and a strictly positive prior $\mu \in \Gamma = \Delta(\Omega) \gg 0$, a finite prize space X, a finite set of actions $\mathcal{A}$, and a finite set of frames F. The decision maker (DM) receives known prize $x(a, \omega) \in X$ when action $a \in \mathcal{A}$ is chosen in state $\omega \in \Omega$. As in Bernheim and Rangel (2009), the prize space can accommodate lotteries). In each decision problem, the DM faces a set of actions $A \neq \emptyset \subset \mathcal{A}$ and a frame $f \in F$, and these problems are indexed by an integer G.

Definition 1 *A* ***framed decision problem*** *is defined as* $G = (A(G), f(G))$. *The set of framed decision problems comprise all choice set-frame combinations,*

$$\mathcal{G}^* = \{G = (A(G), f(G))\,|A(G) \neq \emptyset \subset \mathcal{A} \text{ and } f \in F\}.$$

As noted by Bernheim and Rangel (2009), there is a great deal of flexibility in each particular application about how to draw the line between the frame and the other elements of the choice architecture. As in that paper, we remain agnostic about where to draw that line and provide the tools to proceed once that distinction is made.

Salant and Rubinstein (2008) and Bernheim and Rangel (2009) instead add frames to a standard problem from the decision-theoretic literature, where the only primitive is the set of prizes. Thus, our extended choice problem is a generalization of the "extended choice problem" of Salant and Rubinstein (2008) and the "generalized choice situation" of Bernheim and Rangel (2009). To see that their approaches are nested here, note that the decision problems they consider can be written as $G = (X(G), f(G))$. When the the prior is degenerate, there is a one-to-one map between A and X, so $A(G)$ can be replaced with $X(G)$.

As noted by Bernheim and Rangel (2009), the prize set can be interpreted to include all incomplete information of the DM, which is suitable when all welfare-relevant states are internal to the DM. We expand on their framework to allow for a richer world where some welfare-relevant states are observable to the policy maker (PM) . A leading case is when choice options are complex, so that they are composed of characteristics that agents may not fully internalize, but can be recorded by the PM. For example, imagine that employees can choose a "2040 target date" fund for their retirement portfolios, which has many components. Because of this complexity, employees may be uncertain about what components they will get if they take the action of choosing the fund, but a policy maker would certainly know them.

Because of its generality, our approach can be used to study the wide range of framing effects that are considered in Salant and Rubinstein (2008) and Bernheim and Rangel (2009).

Example 1 (From Salant and Rubinstein 2008) *One of the prizes is designated as the default prize. The collection of frames F is the set of all possible prizes $x \in X$. An extended choice problem is a pair (X, x).*

Example 2 (Extension under new framework) *One action is designated as the default action, but the DM is uncertain about the exact prize they will get for taking each action. A state ω indicates the actual prize obtained x from taking action a. The decision makers initial uncertainty is summarized by a distribution μ over states Ω. A framed decision problem is then a pair (A, a) given (Ω, μ, X, x).*

Example 3 (From Salant and Rubinstein 2008) *The DM has a time limit t to make a choice. An extended choice problem is a pair (X, t).*

Example 4 (Extension under new framework) *The DM has a time limit, which has a strong impact because he or she is also uncertain about the prize associated with each action. A framed decision problem is then a pair (A, t) given (Ω, μ, X, x).*

Note that the frame is separately specified from prior probabilities. This is a consequence of assuming that frames do not communicate decision-relevant information. The model can readily be extended to cases where frames do communicate decision relevant information by correspondingly enriching the notion of a choice set or by allowing decision makers to be uncertain about the frame itself (as in Caplin and Martin 2012).

3.2 The Policy Maker's Data

From now on we consider a PM who observes a set of framed decision problems. Unlike much of the literature on framing effects, we do not require the PM to observe all possible framed decision problems. Instead, we impose no restrictions on the observed subset of framed decision problems $\mathcal{G} \subset \mathcal{G}^*$. However, note that both the frame and the action set can vary across these framed decision problems, which does mirror the existing literature on framing effects.

We now specify the behavioral data set that the PM is assumed to gather for each framed decision problem. As is implicitly assumed in the literature on information design, we assume that both states and actions are fully observed by the PM, and that the PM observes the full distribution of actual state realizations and action choices.

In formal terms, our behavioral data set is therefore an extension of state-dependent stochastic choice (SDSC) data of CM.[7] An SDSC data set $P \in \mathcal{P}$ specifies probabilities $P(a, \omega) \geq 0$ over all possible state-action combinations consistent with the prior,

$$P \in \mathcal{P} \Longleftrightarrow \sum_{a \in \mathcal{A}} P(a, \omega) = \mu(\omega).$$

The unconditional action probabilities are given by $P(a) = \sum_{\omega \in \Omega} \mu(\omega) P(a|\omega)$, and the set of all chosen actions is $B(P) = \{a \in \mathcal{A} | P(a) > 0\}$.

Definition 2 *A* ***framed SDSC data set*** P_G *is the SDSC data set* P *that corresponds to decision problem* $A(G)$ *under frame* $f(G)$*. Further, a* ***framed behavioral data set*** $\mathcal{P}_{\mathcal{G}}$ *is the set of all framed SDSC data sets for an observed set of decision problems* $\mathcal{G}$*.*

Clearly, all choices in P_G much be consistent with G, so that $B(P_G) \subset A(G)$. For notational simplicity, we define $B(G) = B(P_G)$.

[7] This can be interpreted as the PM watching the DM face this decision infinitely often. In practice, one might apply a model of this form to a population rather than an individual, as in the literature on discrete choice following McFadden (2005).

Definition 3 *Given P_G, we define the* ***conditional prize probabilities*** *given states respectively as:*

$$P_G(x|\omega) \equiv \sum_{\{a \in B(G) | x(a,\omega)=x\}} P_G(a|\omega).$$

We define also the unconditional prize probabilities as $P_G(x) = \sum_{\omega \in \Omega} P_G(x,\omega)\mu(\omega)$.

Using these objects, we can cleanly define a framing effect within our framework. This general definition of framing effects allows for any effect of the frame on the resulting pattern of prize choice.

Definition 4 *$P_\mathcal{G}$ displays a* ***framing effect*** *if there exist $G \neq H \in \mathcal{G}$ with $A(G) = A(H)$ and such that,*

$$P_G\left(x|\omega\right) \neq P_H\left(x|\omega\right),$$

for some $x \in X$ and $\omega \in \Omega$.

This definition differs from ones in the existing literature on two dimensions. First, it is based on stochastic choice, not deterministic choice. Second, it allows for state-dependence in framing effects, which is new to our framework. Without this dependency, a framing effect may be missed because an increase in the choice of a prize in one state is offset by a decrease in another state.

3.3 Expanded Example

To illustrate how our framework can be used to study framing effects, we expand on an earlier example (example 4). We will return to this expanded example in section 6.

Example 5 (Two Time Limits) *The DM faces a time limit when making choices, and there are two such frames: a long time t to make a choice and short time s to make a choice. The DM decides between choosing an available plan (action a), which is of uncertain quality, and one of two outside options b or c, which are of certain quality. The plan is equally likely to be of type "g" (state g) or type "ng" (state ng). The DM always chooses between a plan and an outside option, so the possible choice sets are $B = \{a, b\}$ or $C = \{a, c\}$. Thus, the set of possible framed decision problems is $\{Bt \equiv G_1 = (B, t),\ Ct \equiv G_2 = (C, t),\ Bs \equiv G_3 = (B, s),\ Cs \equiv G_4 = (C, s)\}$. The*

SDSC data set in frame t is:

$$\begin{aligned} P_{Bt}(a,g) &= P_{Ct}(a,g) = P_{Bt}(b,ng) = P_{Ct}(c,ng) = 0.4; \\ P_{Bt}(a,ng) &= P_{Ct}(a,ng) = P_{Bt}(b,g) = P_{Ct}(c,g) = 0.1; \end{aligned}$$

The SDSC data in frame s is:

$$\begin{aligned} P_{Bs}(a,g) &= P_{Bs}(a,ng) = 0.5; \\ P_{Cs}(a,g) &= P_{Cs}(c,ng) = 0.3; \\ P_{Cs}(c,g) &= P_{Cs}(a,ng) = 0.2. \end{aligned}$$

This “Two Time Limits” example exhibits a number of framing effects. For instance, when the time limit is short, the DM is more likely to choose the plan over outside option b when the plan is of type g.

In section 6, we will use this example to demonstrate how our approach can be used to answer a number of important welfare questions. First, we will ask if the data in this example can be rationalized by a underlying frame-independent utility function, and if so, what is the full set of such utility functions. The answer to the former will be yes, and we will learn that the data tells us that the “g” type plan is revealed preferred to the “ng” type plan. In other words, the “g” type plan is the “good” plan, and the “ng” type plan is the “not good” plan.

Second, we will ask if one can rank the two frames (long time limit vs. short time limit) in terms of total expected utility and informativeness. Instead of assuming that the longer time limit leaves individuals more informed, we will use the data to reach a conclusion. It could be that the short time limit helps people to focus their attention, which would be missed if we just followed our intuition. Given this data, the short time limit frame s will be “revealed” to make options more difficult to understand.

Third, we will ask if one can account for this data with some theory of rational choice in the face of costly information, and if so, what defines rationalizing cost functions. Finally, we will ask if we can make a welfare comparison of these two frames that takes account of both the expected utility from prizes and the informational costs required to achieve that level of expected utility. Once again, the answer to both of these questions will be yes.

3.4 Information Strategies and Generated Data

Our first question concerns how to identify when a data set is consistent with optimization of a frame-independent utility function given some form of frame-dependent information. As in Kamenica and Gentzkow (2011) and Caplin, Dean, and Leahy (2017) (hereafter CDL), we follow the information design literature and define an information strategy in terms of the resulting posteriors and their implications for choice. The set Π is the set of simple distributions over posteriors consistent with the prior,

$$\Pi \equiv \left\{ \pi \in \Delta(\Gamma) | \sum_{\gamma \in \Gamma(\pi)} \gamma \pi(\gamma) = \mu \right\},$$

where $\Gamma(\pi)$ specifies the finite set of posteriors that have strictly positive probability in π. An information strategy $\lambda = (\pi_\lambda, q_\lambda) \in \Lambda$ comprises $\pi_\lambda \in \Pi$ and mixed action strategy, $q_\lambda : \Gamma(\pi_\lambda) \to \Delta(\mathcal{A})$, with $B(\lambda)$ the chosen actions with $q_\lambda(a|\gamma) > 0$ for some $\gamma \in \Gamma(\pi_\lambda)$.

To translate an arbitrary strategy into its observable counterpart in SDSC data, $\mathbf{P}_\lambda$, we assume "rational expectations".[8] Given $\lambda \in \Lambda$, a generated SDSC data $\mathbf{P}_\lambda$ specifies probabilities over all possible state-action combinations:

$$\mathbf{P}_\lambda(a, \omega) = \sum_{\gamma \in \Gamma(\pi_\lambda)} q_\lambda(a|\gamma) \pi_\lambda(\gamma) \gamma(\omega) \geq 0 \ .$$

The unconditional action probabilities are $\mathbf{P}_\lambda(a) = \sum_{\gamma \in \Gamma(\pi_\lambda)} q_\lambda(a|\gamma) \pi_\lambda(\gamma)$ with $\sum_{a \in B(\lambda)} \mathbf{P}_\lambda(a) = 1$.

We are particularly interested in how the frame impacts the information strategy, and we will look to recover this from choice data. To that end we let $\Lambda(G)$ denote the strategies consistent with framed decision problem G,

$$\Lambda(G) \equiv \{\lambda \in \Lambda | B(\lambda) \subset A(G)\} .$$

Thus, for all $G \in \mathcal{G}$ we will look to recover $\mathbf{P}_\lambda$ in which $\lambda \in \Lambda(G)$.

[8] As in CDL, rational expectations involves common probability assessments (for commonly observed unconditional state and action choice probabilities) between PM and DM, which also makes the prior itself observable.

4 Framed Information Representations

In this section, we identify conditions on the data for an underlying frame-independent utility function to rationalize it. This goal is central to framing effects literature, appearing in Salant and Rubinstein (2008), Bernheim and Rangel (2009), and Rubinstein and Salant (2011). Here we extend existing results from the information design literature to provide a comparable answer within our framework.

4.1 Representation

We assume from now on that all frames and actions feature in at least one decision problem, and that all prizes are chosen at least once. Were this to be false, one would trim surplus elements and apply the analysis that follows to those that remain. To avoid triviality, we insist also that the utility function not rank all prizes as indifferent. With this, we normalize to class $\mathcal{U}$ in which the utility of the best prize is 1 and the worst is 0.

$$\mathcal{U} \equiv \left\{ U : X \to \mathbb{R} | \max_{x \in X} U(x) = 1 \text{ and } \min_{x \in X} U(x) = 0 \right\}.$$

We can now define precisely what it means for the choice data to be rationalized by $U \in \mathcal{U}$. While we will be insistent that the utility function is fixed across frames, we allow information to vary with the frame. This will require us to guarantee existence of an information strategy that is consistent with the data for all $G \in \mathcal{G}$. When $P_{\mathcal{G}}$ has a representation, we define $\mathcal{U}^{IR}(P_{\mathcal{G}}) \subset \mathcal{U}$ as the class of rationalizing cost functions.

Definition 5 *$U \in \mathcal{U}$ provides a* ***framed information representation*** *(an* ***F-IR****) for $P_{\mathcal{G}}$ if for all $G \in \mathcal{G}$ there exist strategies $\lambda \in \Lambda(G)$ satisfying:*

1. ***Data Matching****: Given $a \in A(G)$ and $\omega \in \Omega$,*

$$\mathbf{P}_{\lambda}(a, \omega) = P_G(a, \omega).$$

2. ***Maximization:*** *Given $\gamma \in \Gamma(\pi_\lambda)$, $a \in B(G)$ with $q_\lambda(a|\gamma) > 0$, and $c \in A(G)$,*

$$\sum_{\omega \in \Omega} \gamma(\omega) U\left[x(a,\omega)\right] \geq \sum_{\omega \in \Omega} \gamma(\omega) U\left[x(c,\omega)\right]. \tag{1}$$

Given non-constancy of the utility function, note that the maximization inequality is sometimes strict.

4.2 Framed No Improving Action Switches (F-NIAS)

Given the role that posteriors play in the definition of an F-IR, it is important to extract them from the choice data. Following CM we associate with each chosen action the corresponding "revealed posterior". This treats the action as chosen at one and only one posterior which can be inferred from the behavioral data using Bayes' rule. Given $P \in \mathcal{P}$ and $a \in B(P)$, a revealed posterior $\bar{\gamma}_P^a$ for state ω is:

$$\bar{\gamma}_P^a(\omega) = \frac{P(a,\omega)}{P(a)},$$

with $\Gamma_P = \cup_{a \in B(P)} \bar{\gamma}_P^a$. For notational simplicity, we denote $\bar{\gamma}_G^a = \bar{\gamma}_{P_G}^a$ and $\Gamma_G = \Gamma_{P_G}$.

The behavioral restrictions for an F-IR represent a straightforward extension of the "No Improving Action Switches" (NIAS) inequalities introduced in CM, which are used by Caplin and Dean (2015), CDL, and Chambers, Liu, and Rehbeck (2017) to characterize models of rational attention. NIAS insists that all chosen actions maximize expected utility at the corresponding revealed posterior.

Definition 6 *Given $P_{\mathcal{G}}$, $U \in \mathcal{U}$ satisfies the* ***Framed No Improving Action Switches (F-NIAS) inequalities*** *if, for all $G \in \mathcal{G}$ and all $a \in B(G)$ and $c \in A(G)$,*

$$\sum_{\omega \in \Omega} \bar{\gamma}_G^a(\omega) U\left[x(a,\omega)\right] \geq \sum_{\omega \in \Omega} \bar{\gamma}_G^a(\omega) U\left[x(c,\omega)\right].$$

Since we have insisted that all prizes are observed in the choice data, non-constancy of the utility function implies that this inequality is sometimes strict.

Theorem 1 shows that F-NIAS characterizes F-IR. This is a direct extension of the main result of CM, so is given here without proof. All that is required to extend their proof is to add frame subscripts to each choice set. This extension is particularly useful for the study of framing effects because F-NIAS crisply summarizes the conditions for the existence of a frame-independent utility function that rationalizes choices from framed decision problems.

Theorem 1 *$U \in \mathcal{U}^{IR}(P_{\mathcal{G}})$ if and only if the F-NIAS inequalities hold for U given $P_{\mathcal{G}}$.*

As with the frame-independent preference relation identified in Salant and Rubinstein (2008) and Bernheim and Rangel (2009), any U produced here is a candidate for performing a welfare analysis.

5 Ranking Frames

Framing is becoming an increasingly popular tool for policy makers, but policy makers may be concerned about whether they are selecting the welfare optimal frame from a set of possibilities. We now show two ways to rank frames based on utility functions and revealed posteriors identified within our framework.

5.1 Ranking Frames with Utility

When an F-IR exists, it is natural to seek to rank frames according to the expected utility that they generate. First we define the expected utility of choosing from action set $A(G)$ given frame $f(G)$.

Definition 7 *Given $P_{\mathcal{G}}$, $U \in \mathcal{U}^{IR}(P_{\mathcal{G}})$, and $G \in \mathcal{G}$, the expected utility $E_G(U)$ associated with the observed data can be computed as,*

$$E_G(U) = \sum_{a \in B(G)} \sum_{\omega \in \Omega} P_G(a, \omega) U \left[x(a, \omega) \right].$$

Since there may be multiple functions U that satisfy F-NIAS, we will take a cautious "Pareto" approach, in the style of Bernheim and Rangel (2009) and Benkert and Netzer (2016), and only rank frames when all consistent utility orderings are in agreement. As in the definition of a "successful nudge" proposed in Benkert and Netzer (2016), we will also look for agreement across decision problems.

Definition 8 *Given $P_{\mathcal{G}}$ with an F-IR, frame f yields* ***weakly (strictly) higher expected utility*** *than than frame f' if, for all $G \neq G' \in \mathcal{G}$ with $A(G) = A(G')$, $f(G) = f$, and $f(G') = f'$,*

$$U \in \mathcal{U}^{IR}(P_{\mathcal{G}}) \Longrightarrow E_G(U) \geq (>) E_{G'}(U).$$

Like the ranking of frames presented in Benkert and Netzer (2016), our ranking looks for consistency in the ranking across choice sets and is a partial order if $\mathcal{G}$ includes all combinations of actions and frames. Also, like the ranking presented in Benkert and Netzer (2016), it will often be incomplete due to the conservative nature of the Pareto approach.[9] In particular, there may be distinct utility functions $U, U' \in \mathcal{U}^{IR}(P_{\mathcal{G}})$ that change the implied comparison of utility across frames.

5.2 Ranking Frames with Information

A policy maker may want decision makers to be as informed as possible when making choices, so we also provide a ranking based on informativeness. Rather than assume that one frame makes decision options easier to understand, we use the choice data to indicate this is true. Of course, there is also a link between a frame being informative and a frame having high expected utility. The seminal result of Blackwell (1953) links differences in information with differences in utility.

To implement this idea and rank frames in terms of their informativeness, we identify a "revealed posterior-based strategy" for each $G \in \mathcal{G}$. Given $P \in \mathcal{P}$, a revealed posterior-based information strategy $\boldsymbol{\lambda}(P) = (\boldsymbol{\pi}_P, \mathbf{q}_P)$ for $\gamma \in \Gamma(P)$ is:

$$\begin{aligned} \boldsymbol{\pi}_P(\gamma) &= \sum_{\{a \in B(P)) | \bar{\gamma}_P^a = \gamma\}} P(a); \\ \mathbf{q}_P(a|\gamma) &= \begin{cases} \frac{P(a)}{\boldsymbol{\pi}_P(\gamma)} & \text{if } \bar{\gamma}_P^a = \gamma; \\ 0 & \text{if } \bar{\gamma}_P^a \neq \gamma. \end{cases} \end{aligned}$$

Caplin and Dean (2015) (hereafter CD) show that, while there is a multiplicity of strategies that

[9]Bouacida and Martin (2017) do not find the Pareto approach overly demanding in two empirical applications.

could have generated any SDSC data, this is the unique least Blackwell informative strategy that could have generated the data.

We use short-hand for the corresponding strategy revealed in the data set for each $G \in \mathcal{G}$: $\boldsymbol{\lambda}(G) = (\boldsymbol{\pi}_G, \mathbf{q}_G) \equiv \boldsymbol{\lambda}(P_G)$. With this strategy in place we can compare the information associated with different frames. In fact we can use the Blackwell ordering on revealed information structures to order frames by their informativeness.

Definition 9 *Given $P_{\mathcal{G}}$ with an F-IR, frame f is* ***weakly (strictly) more informative*** *than frame f' if, for all $G \neq G' \in \mathcal{G}$ with $A(G) = A(G')$, $f(G) = f$, and $f(G') = f'$, the distribution of posteriors $\boldsymbol{\pi}_G$ is always weakly (strictly) more Blackwell informative than the distribution of posteriors $\boldsymbol{\pi}_{G'}$.*

This ranking defines a sense in which one frame may make it easier than another for the DM to understand the prizes available. This provides a partial order if $\mathcal{G}$ includes all combinations of actions and frames.

This ranking is also useful because it provides an alternative way to rank frames based on utility. When the comparison of information strategies is revealing enough, we know already from Blackwell (1953) that conclusions about utility comparisons may follow from observations about information alone. In fact, it is immediate from Blackwell's theorem that given $P_{\mathcal{G}}$ with an F-IR, if frame f is more informative than frame f', it also yields higher expected utility.

Of course, it may be too much to ask for such a strong condition to hold. When it does not, the ranking in terms of utility may still hold provided the F-NIAS conditions sufficiently restrict the class of permissible utility functions.

6 Framed Costly Information Representations

A holistic view of the welfare consequence of framing must take account of costs of information acquisition that may differ between frames. For example, if information search incurs cognitive costs (as studied in Caplin, Dean, and Martin 2011), and some frames make information search more difficult, then DMs may have different information costs in different frames, both because they can change the amount that they search between frames and because search is more costly in certain frames.

As a result, a ranking of frames using the expected utility of prizes alone (such as the ranking of Benkert and Netzer (2016) and the rankings presented in the previous section) might not match one that uses expected utility net of information costs. However, due to the challenges in incorporating information costs into the welfare assessment of frames, no such ranking exists in the literature. In this section, we provide just such a method for ranking frames based on information costs and expected utility net of information costs.

6.1 Frame-Dependent Information Costs

As in much of the literature on costly information, we analyze a DM seeking to maximize expected utility net of additively separable information costs (for example, see Gentzkow and Kamenica 2014). We assume that DMs can choose the amount of information to acquire and that they can choose to acquire no information at a cost of zero. We assume weak respect for the Blackwell ordering and allow also for the possibility that some distributions of posteriors are infeasible by setting their costs to infinity. $\mathcal{K}$ is the set of all such information cost functions $K : \Pi \to \bar{\mathbb{R}}$.

We are interested in whether or not observed data is consistent with a frame-dependent cost function that can vary across frames, but does not vary within a frame. To formalize this, we assume that decisions are made to optimally balance the value of information against its cost. For an agent with cost function $K \in \mathcal{K}$ and utility function $U : X \to \mathbb{R}$, the value of any strategy $\lambda \in \Lambda(G)$ is computed based on additive separability of prize utility and information costs:

$$V(\lambda|U, K) \equiv EU(\lambda) - K(\pi_\lambda);$$

where,

$$EU(\lambda) \equiv \sum_{\gamma \in \Gamma(\pi_\lambda)} \sum_{a \in B(\lambda)} \pi_\lambda(\gamma) q_\lambda(a|\gamma) \sum_{\omega \in \Omega} \gamma(\omega) U\left[x(a, \omega)\right].$$

The value function and corresponding optimal strategies for $G \in \mathcal{G}$ are then defined as:

$$\begin{aligned} \hat{V}(G|U, K) &\equiv \sup_{\lambda \in \Lambda(G)} V(\lambda|U, K); \\ \hat{\Lambda}(G|U, K) &\equiv \left\{ \lambda \in \Lambda(G) | V(\lambda|U, K) = \hat{V}(G|U, K) \right\}. \end{aligned}$$

We now define what it means for optimal strategies with frame-based costs to explain the data. As with an F-IR. we will insist that the utility function is fixed across frames, while we will allow the cost function to vary with the frame. However we will insist that the cost function is fixed within a frame. Since the logic works frame-by-frame, it is important to identify all $G \in \mathcal{G}$ with common frame $f \in F$, which we denote as $\mathcal{G}(f) = \{G \in \mathcal{G} | f(G) = f\}$.

Definition 10 *$U \in \mathcal{U}$ and $\{K_f \in \mathcal{K} | f \in F\}$ provide a* ***framed costly information representation (F-CIR)*** *of $P_{\mathcal{G}}$ if for all $f \in F$ and $G \in \mathcal{G}(f)$, there exists $\lambda \in \hat{\Lambda}(G|U, K_f)$ s.t. $\mathbf{P}_\lambda = P_G$.*

When $P_{\mathcal{G}}$ has an F-CIR, we define $\mathcal{U}^{CIR}(P_{\mathcal{G}}) \subset \mathcal{U}$ as the class of rationalizing utility functions. We define $\mathcal{K}_f^{CIR}(U) \neq \emptyset \subset \mathcal{K}$ for all $f \in F$ as the corresponding rationalizing cost functions for a given $U \in \mathcal{U}^{CIR}(P_{\mathcal{G}})$.

6.2 Framed No Improving Attention Cycles (F-NIAC)

It is immediate from this definition that $\mathcal{U}^{CIR}(P_{\mathcal{G}}) \subset \mathcal{U}^{IR}(P_{\mathcal{G}})$, since Theorem 1 implies that any utility function in $\mathcal{U} \backslash \mathcal{U}^{IR}(P_{\mathcal{G}})$ fails to admit an optimal strategy that matches the data. Optimality requires the F-NIAS inequalities to hold, which we know to be equivalent to an F-IR. We now identify the additional condition that determines the utility functions in $\mathcal{U}^{CIR}(P_{\mathcal{G}})$, which is an extension of the "No Improving Attention Cycles" (NIAC) condition from CD. To specify this condition, we specify the maximum utility that is available for an arbitrary combination of an action set A and an SDSC data set P. Given $U \in \mathcal{U}$, $A \neq \emptyset \subset \mathcal{A}$, and $P \in \mathcal{P}$,

$$\hat{U}(A, P) \equiv \sum_{b \in B(P)} P(b) \max_{a \in A} \sum_{\omega \in \Omega} \bar{\gamma}_P^b(\omega) U\left[x(a, \omega)\right].$$

Definition 11 *Given $P_{\mathcal{G}}$, the* ***Framed No Improving Attention Cycles (F-NIAC) inequalities*** *hold for $U \in \mathcal{U}$ if, given any $f \in F$ and sequence of $G_m \in \mathcal{G}(f)$ indexed by $1 \leq m \leq M$ where $G_1 = G_M$,*

$$\sum_{m=1}^{M-1} \hat{U}(A(G_m), P_{G_m}) \geq \sum_{m=1}^{M-1} \hat{U}(A(G_m), P_{G_{m+1}}).$$

The F-NIAC inequalities rule out switching information strategies across decision problems with the same frame in a manner that raises total utility. Our second result, a straightforward extension of the main result in CD, is that satisfaction of these inequalities – in addition to the F-NIAS inequalities – is necessary and sufficient for an F-CIR.

Theorem 2 *Given $P_{\mathcal{G}}$, $\mathcal{U}^{CIR}(P_{\mathcal{G}})$ comprises the subset of $\mathcal{U}^{IR}(P_{\mathcal{G}})$ for which the F-NIAC inequalities hold.*

Similar to theorem 1, this is a direct extension of the main result in CD, so is given here without proof. All that is required to extend their proof is to add frame subscripts to each choice set. At first their result may appear distinct, since CD take the utility function as known and then provide conditions for existence of a rationalizing cost function. In contrast, in this paper we are looking to identify utility functions also. However this distinction does not affect the proof, since the utility function we are testing is fixed as $U \in \mathcal{U}^{IR}(P_{\mathcal{G}})$ that satisfies the F-NIAC inequalities by the time we apply the NIAC conditions. Note that there is no link between distinct frames, so one can find the set of rationalizing cost functions frame-by-frame.

Theorems 1 and 2 show that F-NIAC and F-NIAS together provide conditions for the existence of a frame-independent utility function that rationalizes choices from framed decision problems that involve information costs. It also implies the existence of cost functions for each frame, which can be used to determine the information costs implied by the choices made.

6.3 Expanded Example: Frame-Independent Utility

We now return to the "Two Time Limits" example introduced in section 4 and show that both the F-NIAS and F-NIAC conditions are satisfied for that example. As a consequence, there exists a frame-independent utility function and frame-dependent information costs that rationalize the data. We will go further by also using the restrictions to pin down the set of admissible utility functions.

The first key point is to note that the revealed posteriors in the long time limit frame t are $(0.8, 0.2)$ when action a is taken (the "plan" is chosen) and $(0.2, 0.8)$ when actions b or c are taken (the outside option is chosen). This implies directly that if there is any non-trivial utility function, it must be that action a offers strictly higher utility in state g than in state ng,

$$1 = U(g) > U(ng) = 0.$$

The revealed posterior imply that F-NIAS in this frame requires,

$$U(b), U(c) \in [0.2, 0.8].$$

The F-NIAS inequalities for the short time limit frame impose two more conditions. First, $U(b) \leq 0.5$ from the fact that a is chosen over b, and second, $U(c) \in [0.4, 0.6]$ from the fact that the revealed posterior in choice set C is $(0.6, 0.4)$ when action a is taken and $(0.4, 0.6)$ when action c is taken.

For the remaining analysis, it is useful to note that expected utility is:

$$E_{Bt}(U) = 0.4 + 0.5U(b); \quad E_{Ct}(U) = 0.4 + 0.5U(c);$$
$$E_{Bs}(U) = 0.5; \quad E_{Cs}(U) = 0.3 + 0.5U(c).$$

The F-NIAC inequalities have no bite in the long time limit frame, since the distribution of revealed posteriors $\boldsymbol{\pi}$ is the same in decision problems B and C. In the short time limit frame, they say that the sum of the actual realized levels of EU with the strategies that were revealed is at least as high as if they were switched. The switch of the inattentive strategy in B to C would yield $\max[0.5, U(c)]$, while the converse switch would yield $\max[0.3 + 0.5U(b), 0.5]$. Hence F-NIAC implies,

$$0.3 + 0.5U(c) \geq \max[0.3 + 0.5U(b), 0.5] + \max[0, U(c) - 0.5]$$

The only new restrictions this imposes are $U(c) \geq U(b)$ and $U(b) + U(c) \leq 1$.

As a consequence, the feasible utility functions are those with $1 = U(g) > U(ng) = 0$, $U(b) \in [0.2, 0.5]$, $U(c) \in [\max[0.4, U(b)], 0.6]$, and $U(b) + U(c) \leq 1$. What do these restrictions tell us about welfare? First, the type “g” plan is welfare improving over the type “ng” plan. Second, the type “g” plan is welfare improving over receiving *both* outside options at once. Third, outside option c is weakly welfare improving over outside option b.

6.4 Ranking Frames Net of Information Costs

When an F-CIR exists, the ideal is to make welfare comparisons by subtracting costs of information from expected utility (net expected utility). For this purpose, we define the following relation.

Definition 12 *Given $P_{\mathcal{G}}$ with an F-CIR, frame f yields* ***weakly (strictly) higher net expected utility*** *than frame f' if given $U \in \mathcal{U}^{CIR}(P_{\mathcal{G}})$, $K_f \in \mathcal{K}_f^{CIR}(U)$, and $K_{f'} \in \mathcal{K}_{f'}^{CIR}(U)$,*

$$U(G) - K_f(\boldsymbol{\pi}_G) \geq (>) U(G') - K_{f'}(\boldsymbol{\pi}_{G'}),$$

for all $G \neq G' \in \mathcal{G}$ with $A(G) = A(G')$, $f(G) = f$, and $f(G') = f'$.

6.5 Expanded Example: Ranking Frames

Based on the restrictions that F-NIAS and F-NIAC place on the frame-independent utility function, it is direct that frame t yields higher expected utility than s, since

$$E_{Bt}(U) = 0.4 + 0.5U(b) \geq 0.5 = E_{Bs}(U);$$
$$E_{Ct}(U) = 0.4 + 0.5U(c) > 0.3 + 0.5U(c) = E_{Cs}(U).$$

This draws from the fact that the distribution of posteriors under t is Blackwell more informative in both cases. Thus, frame t is also more informative than frame s.

We now round out our analysis of the example by showing that the long time limit frame yields higher net expected utility than the short time limit frame. In effect there are only two revealed posterior distributions, and it is these information costs we need to consider. We let K_s be the cost of the revealed distribution with s in C which involves $(0.4, 0.6)$ and $(0.6, 0.4)$ with equal probability and K_t be the cost of the revealed distribution with t which involves $(0.2, 0.8)$ and $(0.8, 0.2)$ with equal probability.

For set B note that with frame t the option of picking a always was available and free, so that,

$$E_{Bt}(U) - K_t = 0.4 + 0.5U(b) - K_t \geq 0.5 = E_{Bs}(U);$$

and that $K_t \leq 0.5U(b) - 0.1$. We are left to consider set C, in which we know that $E_{Ct}(U) - E_{Cs}(U) = 0.1$. Hence the comparison follows provided costs are higher for t. Specifically,

$$K_t - K_s \leq 0.1.$$

This follows from the fact that it cannot be worthwhile in Bs to switch to the strategy used in Cs and then optimize from there. Hence,

$$\max\{0.3 + 0.5U(b), 0.5\} - K_s \leq 0.5;$$

or,

$$K_s \geq \begin{cases} 0 \text{ if } U(b) \in [0.2, 0.4]; \\ 0.5U(b) - 0.2 \text{ if } U(b) \in [0.4, 0.5]. \end{cases}$$

Either case is fine and validates $K_t - K_s \leq 0.1$. This is clear if $U(b) \in [0.2, 0.4]$, since in this case it is direct that $K_t \leq 0.5U(b) - 0.1 \leq 0.1$. If $U(b) \in [0.4, 0.5]$, then the fact that $K_t \leq 0.5U(b) - 0.1$ and that $K_s \geq 0.5U(b) - 0.2$ yields directly that $K_t - K_s \leq 0.1$.

From these restrictions on utility U and the costs of information K_t and K_s, which are generated by F-NIAS and F-NIAC, it is direct that the long time limit frame yields higher net expected utility than the short time limit frame.

7 Concluding Remarks

We have introduced a model of framing effects that departs from the literature by having frames impact choice by changing beliefs, and as in the information design literature, we summarize beliefs using information structures over observable states. We hope this model illustrates the promise of using tools from information theory to understand the impact of nudges and other policy interventions.

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