THE DUAL-PROCESS DRIFT DIFFUSION MODEL: EVIDENCE FROM RESPONSE TIMES

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We introduce a model of response time and choice that borrows from two distinct psychological traditions. As in dual-process models, rapid (automatic) decisions are qualitatively different from considered decisions. As in the drift diffusion model, delayed (considered) decisions occur when confidence hits a threshold level. We conduct a simple experiment in which our hybrid model matches key properties of the data. As our model predicts, decision times are bimodal, automatic decisions are of far lower quality than considered decisions, and automatic decisions are more prevalent when prior information improves, thereby raising their quality. (JEL D83, D87, C91)

I. INTRODUCTION

Economists are increasingly interested in connecting choice behavior with response time.1 One source of inspiration for the economics literature is the “dual-process” approach, which draws on a psychological tradition of contrasting rapid “automatic” decisions with more time-consuming “considered” decisions.2 For instance, Rubinstein (2007) posits that in some matrix games “choices that require more cognitive activity will result in longer response times than choices that involve an instinctive response.”

A second source of inspiration for the economics literature is the drift diffusion model (DDM) introduced by Ratcliff (1978), which maps the gradual accretion of evidence up to a decision-inducing threshold.3 Clithero and Rangel (2013) show that the DDM, when calibrated with response times, is better at predicting out-of-sample choices than the standard Logistic approach.

We introduce a dual-process DDM that borrows from both psychological traditions. As in the dual-process approach, individuals make either automatic decisions or considered decisions. Automatic decisions are rapid and based only on general features of the choice environment. Considered decisions involve waiting for a threshold decision quality to be achieved, as in the DDM. We follow much of this literature in treating the parameters of the DDM as fixed for a given level of perceptual difficulty. Hence, choice accuracy and the distribution of response times do not change unless the perceptual difficulty changes.

The key innovation in our model is that individuals choose whether to make automatic or considered decisions.4 In making this choice, they face the following trade-off: while making a considered decision can produce better choices, and Rangel (2013) show that the DDM, when calibrated with response times, is better at predicting out-of-sample choices than the standard Logistic approach.

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WEAI

ABBREVIATIONS

CES: Central Executive System

DDM: Drift Diffusion Model

RDV: Relative Decision Value

1. Fehr and Rangel (2011) and Spiliopoulos and Ortmann (2014) provide valuable surveys of portions of this literature.
3. Remarkable neuroscientific findings (e.g., Shadlen and Newsome 2001) have spurred application and development of this model to economic decisions (Clithero and Rangel 2013; Fudenberg, Strack, and Strzalecki 2015; Krajbich, Armel, and

4. This distinguishes our model from the dual-process drift model of Alos-Ferrer (2015) in which the process for determining automatic and considered decisions is exogenous.
it also involves an “attentional” cost. In our model, this cost is stochastic, but broadly follows idiosyncratic features of the subject and the situation. For example, the distribution of costs shifts to the right when the decision maker is tired or evidence becomes more difficult to process. The individual decides which type of decision to make after a quick appraisal of the realized level of such costs.

Our model is designed for settings in which considered decisions involve substantial attentional costs. While this may not be the case in traditional applications in psychology (quick perceptual tasks), it is certainly true for most economic applications. Thus, our dual-process extension may prove useful in applying the DDM to a range of economic choices.

We conduct an experiment the results of which validate our modeling strategy. Subjects have available three actions, one and only one of which yields a prize, but determining which is best requires considerable cognitive effort. Because prior beliefs directly impact the value of automatic decisions, our central point of treatment variation is the probability each action is best.

3. Furthermore, as the model predicts, the quality of considered decisions is essentially independent of prior beliefs and of the experimental round. Decision makers are correct approximately 66% of the time in each experimental treatment, regardless of how informative is the prior and regardless of the round.

4. As the model predicts, considered decisions are of significantly higher quality than automatic decisions.

5. As the model predicts, automatic decisions are more prevalent when the prior is more informative and in later experimental rounds as subjects get increasingly tired.

In Section II, we introduce our dual-process model. In Section III, we introduce our experiment and define automatic and considered decisions. In Section IV, we match model to data. Section V concludes.

II. THE DUAL PROCESS DDM

A. The Standard DDM

The standard DDM models a subject facing two possible options, such as choosing left or choosing right. These choices give rise to the (known) good prize as opposed to the (known) bad prize with some prior probability. The decision is made non-trivial only by the fact that the evidence on which option yields the good prize is hard to process. In typical psychological experiments, the decision maker (often a monkey) is presented with a flow of evidence that takes effort to process. What is recorded in the experiment is the joint distribution of decision time and decision quality: how long the decision takes, and conditional on its length, how likely is the correct option to be chosen.

The DDM specifies a precise mechanism by which the subject arrives at a decision, and uses it to produce predictions about the resulting experimental data. Specifically, it is assumed that there is some subjective state variable $X(t)$ whose sign and size reflects the relative probability that the good prize is on one side and not the other. In the DDM’s simplest form, the initial condition is $X(0) = 0$ and in each period the variable drifts in the direction of the truth with known drift rate $\mu$ and error $\epsilon(t)$. Hence, when choosing left is in fact the better
choice and $X(t)$ reflects the relatively probability the good prize is on the left, the state evolves as:

$$X(t) = X(t-1) + \mu + \varepsilon(t).$$

The period error is typically assumed to be independently drawn from a normal distribution $N(0, \sigma^2)$.

To close the model in its simplest form, it is assumed that there is some upper bound $B$ and lower bound $-B$ on the absolute value of the subjective state variable, and when the upper bound is hit, the correct choice is made, and when the lower bound is hit, the incorrect choice is made. It is also assumed that the barrier or the drift rate changes with the difficulty in assessing evidence. Hitting the bound is interpreted as the subject reaching a confidence level at which choice can finally be made, and this produces the optimal speed-accuracy trade-off in very particular settings. Figure 1 presents a sample path of $X(t)$ corresponding to a case with drift $\mu = .0005$, error parameter $\sigma = .05$, and bound $B = 1$. In this particular case, note that the correct decision is made.

In economic applications of the DDM, it is the utility differential between prizes that is unknown to the decision maker. For example Krajbich, Armel, and Rangel (2010) illustrate a form of decision paralysis in which a long time is taken to decide between items that are very close in utility. In their DDM, the decision maker takes a long time to resolve this choice because the drift rate becomes small while the decision thresholds remain fixed.

B. The Dual Process DDM

In the example above, if contemplation time is costly, it would be valuable to have available a distinct strategy of rapidly choosing one of the prizes without going through the process of accumulating evidence. Our dual process DDM allows for just such a fast and frugal alternative. It therefore places a limit on the extent to which the decision maker will implement a behavioral rule inappropriate to their actual environment. However, while our dual process DDM takes a step closer to a fully optimal system, it maintains the computational feasibility (and closed form solutions) of a simple DDM.

Technically, we fix a simple DDM strategy as one of two available methods of making a decision. The other method is to make an immediate decision based only on readily available information that does not require attention to the choice alternatives. Before each decision, the decision maker chooses which method to employ by undertaking a cost-benefit calculus.

As mentioned previously, we fix the parameters of the DDM as in much of literature. However, we depart from the literature in assuming that this strategy is attentionally costly, with cost $c$ subtracting from the final expected utility of the chosen option. The cost in expected utility units of the strategy varies from situation to situation according to the cumulative distribution $G(c)$. When facing a particular decision problem, the decision maker gets a quick read on the costs of learning about the choice options in that situation. In practice, these costs may depend on some subjective state of tiredness, being in a hurry, and so on. The decision on whether or not to use the automatic strategy of picking a prior best choice or instead paying attention and picking according to the DDM strategy is based on a direct comparison of the resulting expected utility net of attention costs.

This model can be interpreted as a brain that operates in a hierarchical manner, in the style

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8. This is accomplished by defining the variable $X(t)$ as the log of the probability ratio of one state over the other at time $t$: $\ln(p(t)/(1-p(t)))$. As Gold and Shadlen (2002) and Bogacz et al. (2006) show, this form of behavior can implement the Wald-Wolfowitz likelihood ratio test for optimal stopping in the face of costly information acquisition.

9. While there is evidence that the threshold is not perfectly attuned to each specific realization, one could imagine it being attuned to simple to observe features of the environment—possibly through some evolutionary process.
of Brocas and Carrillo (2008), Alonso, Brocas, and Carrillo (2014), and Cunningham (2014). With such an interpretation, we can imagine a metacognition layer, which Alonso, Brocas, and Carrillo call the “Central Executive System” (CES), that makes decisions after acquiring information from other regions of the brain, which can be costly to activate (in terms of neural firing rates or other cognitive costs). In our model, the center learns the prior at essentially no cost (as in the study by Cunningham 2014), but incurs a much larger cost to activate the region that accrues information as in the DDM. The CES decides whether to activate this region after making a quick appraisal of the realized level of such costs.

We let $E(B) \in (0, 1)$ denote the gross expected utility level associated with accruing information about the choice options. The decision maker gets information on the attention cost in the environment, $c$, and therefore associates expected utility of $E(B) - c$. The alternative to the considered strategy is the automatic strategy. Here the utility is defined based on prior beliefs about the EU associated with each action. We define $\hat{U}_0$ as the expected utility associated with the choice of the action that is judged as optimal based on prior information. It is this expected utility that is associated with the automatic strategy. Which system is used depends on the comparison of these two utilities. If $E(B) - \hat{U}_0 \leq c$, the automatic strategy is chosen because the expected costs of the considered strategy exceed the expected benefits. If $E(B) - \hat{U}_0 > c$, the considered strategy is chosen because the expected costs of the considered strategy is below the expected benefits of choosing the considered strategy over the automatic strategy.

A key simplifying assumption implicit in the above formulation is that the prior has no impact whatsoever on the costs or the benefits of the considered strategy. The primary motivation for making this assumption is analytic simplicity.

Taken together, our model produces the following clear experimental predictions:

1. The first key prediction is that there will be a distinct mass of automatic decisions, which are by definition taken quickly. This prediction is driven by the dual-process part of the model.
2. If the automatic strategy is chosen, expected prize utility is $\hat{U}_0$, hence determined only by the quality of prior information (and not the distribution of attention costs). This prediction is also driven by the dual-process part of the model.
3. If the considered strategy is chosen, expected prize utility is $E(B)$ and is independent of the quality of prior information and the distribution of attention costs. This prediction is driven by the DDM part of the model.
4. If the considered strategy involves a cost, if the considered strategy is chosen, expected prize utility is $E(B)$, which will always be higher than the expected utility $\hat{U}_0$ associated with the automatic strategy. This prediction is driven by the dual-process part of the model.
5. The considered strategy is chosen if and only if the attention cost falls below the utility differential, which occurs with probability $G(E(B) - \hat{U}_0)$. Hence an improvement in the prior information decreases the probability that the considered strategy is chosen, as does a shift out in the subjective costs of implementing the considered strategy. This prediction is driven by both the dual-process and the DDM parts of the model.
6. If the considered strategy is chosen, the joint distribution of decision time and decision quality precisely matches that in the DDM. This prediction is driven by the DDM part of the model.

In the next section, we introduce an experiment that tests our dual process DDM. In light of its simplicity, it is perhaps surprising that the model does as good a job as it does in capturing key features of observed behavior.

Psychologically Plausible Extensions. As mentioned previously, we have intentionally kept our model simple. However, there are many possible extensions of our model that could be made in order to match evidence from the psychology literature. For example, the DDM could be made more complex by allowing the initial point in the drift process $X(0)$ to vary with prior beliefs. In addition, we could allow the boundary $B$ of the DDM to move with attentional costs. However, while consistent with findings in the psychology literature, neither of these extensions is needed to explain our data.

In our present model, cost draws are independent, but not identical because the distribution
can vary with round. Thus, there can be correlation in the probability of choosing an automatic strategy. However, we could imagine a more stark transition—that once decision makers switch to automatic decisions, they never turn back to considered decisions. While such a once-off switch is quite possible in principle, we find evidence against such behavior in our experiment.

III. THE EXPERIMENT

A. Experimental Protocol

We use a task in which the subject has to identify which of three options has the highest numeric value (as in the studies by Caplin, Dean, and Martin 2011 and Caplin and Martin 2011). In this context, a natural measure of decision quality is the proportion of experimental runs in which the “best” option (highest value) was chosen. The only intricacy is that the value of each option is hard to assess because it is expressed in words as the sum of 20 integers. Subjects were told that each of the 20 numbers was drawn independently and uniformly from integers between $-18$ and $18$ (see Appendix S1, Supporting Information, for complete instructions). Figure 2 shows three such options.

Subjects faced three treatments in each session, each of which represents a different probability that the first option is best. In the “33%” treatment, subjects were informed that the first option had a 33% chance of being the highest valued option because a number was drawn randomly from three 1s, three 2s, and three 3s, and the highest valued option was placed in the corresponding position in the list. They were also told that the remaining two options were randomly placed in the remaining positions and that this resulted in an equal chance of the option in each position having the highest value. In the “40%” treatment, subjects were given a similar mechanism, but which resulted in the first option having the highest value with a 40% chance and the other two options each with a 30% chance. Finally, in the “50%” treatment, a similar mechanism gave the first option a 50% chance of having the highest value, and the others a 25% chance.

Each treatment lasted 12 rounds, and the order of these three “blocks” of rounds was randomized. No feedback about performance was provided at any point during the experiment. Also, there was no time limit in any round, and subjects could leave the laboratory whenever they completed all 36 rounds. On average, subjects were in the laboratory for less than 1 hour—the minimum total time was 30 minutes, and the maximum total time was over 2 hours. Before a subject left the laboratory, three of the 36 rounds were randomly selected for payment. In each round selected for payment, if the subject chose the best option, the payment was $8, and if the subject did not choose the best option, the payment was $4. Thus, the maximum total payment was $24, and the minimum total payment was $12. There was no show up fee.

12. In just 4 of 1,908 rounds, more than one option has highest value in that round, so there are two “best” options.

13. Here a number was drawn randomly from four 1s, three 2s, and three 3s.

14. Payment procedures were announced in advance to reduce disruption as subjects departed the laboratory. However, these departures may have introduced some peer effects, which could have contributed to the increase in automatic decisions in later rounds.
Over four sessions, we observed that a total of 53 students completed 1,908 rounds (636 rounds per treatment). All sessions were run in the Center for Experimental Social Science laboratory, and subjects were drawn from the undergraduate population at New York University.

B. Automatic and Considered Decisions

As noted in the Introduction, we measured not only the choices made but also the time taken to arrive at the decision. Across treatments, the average decision time was 43.4 seconds, with a standard deviation of 58.0 seconds. The histogram of decision times across treatments is given in Figure 3.

If very little time was taken on the decision, it appears likely that little attention was paid to the choice options. While care needs to be taken in classifying a decision as having taken “very little time,” there is a clearly a large mass on the left side of the distribution—to the left of the vertical line at 8 seconds. Thus, we label a subject as having made an “automatic” decision in a round if 8 seconds or less was taken to make their decision. Figure 4 provides the histogram of decision times for decisions that take 12 seconds or less.

The results that follow are robust to changes the threshold time, which may be unsurprising given that the majority of “automatic” decisions take just 1 or 2 seconds.

15. In many settings, both in the lab and the field, it is possible to collect information on the time it takes to reach a decision.

IV. MODEL FIT

The large mass of automatic decisions relative to the rest of the response time distribution provides the piece of first experimental evidence consistent with our dual process DDM. Averaging across experiments, almost 40% of all decisions are made very rapidly. For the 60% of decisions that take more than 8 seconds, the mean decision time is fully 69.5 seconds.

A. The Quality of Automatic Decisions

Choice behavior is substantially different in those rounds in which subjects make automatic decisions. Table 1 shows that when decisions are automatic, the first option is chosen far more often than it is the best option. Also, as the likelihood that the first option is best increases, this effect becomes even stronger. When subjects expect the first option to be best 50% of the time, nearly all automatic decisions involve choosing the first option.16 However, when subjects

16. This could also be explained by a drift process that has a starting point which is influenced by prior beliefs.
make considered decisions, they choose the first option in a proportion similar to the rate at which it is best.

Given that subjects making automatic decisions overwhelmingly choose the first option when it is more likely to be best, it is not surprising that they find the best option about as often as the first option is best. In the 33% treatment, they find the best option 37% of the time, in the 40% treatment they find it 39% of the time, and in the 50% treatment they find it 47% of the time. Moreover, this performance is not significantly different across each block of rounds (pairwise comparisons of how often best option chosen, within treatment and between blocks, using two-sided t-tests, 10% significance level).

These findings suggest that automatic decisions reflect no information beyond prior beliefs (independently of the experimental round). On top of this, the probability that the first option is best is not significantly different across each block of rounds (pairwise comparisons of how often best option chosen, within treatment and between blocks, using two-sided t-tests, 10% significance level).

B. The Quality of Considered Decisions

As Table 2 indicates, subjects making considered decisions do not always choose the best option. Instead, they find the best option in roughly two out of three rounds. Looking across blocks of rounds and prior probabilities, there is little variation in this rate. No differences are significant between any two treatments, either overall or between any two blocks of rounds (one or two-sided t-tests, 10% significance level).

These patterns are also consistent with our dual process DDM.

In addition, as the model predicts, this table shows that considered decisions are of significantly higher quality than automatic decisions, regardless of the prior. Even when the prior probability that the first option is best is 50%, the quality of automatic decisions is 47%, whereas the quality of considered decision is 67%.

C. Comparative Statics: Prior Probabilities and Fatigue

As shown in Table 3, the percentage of automatic decisions increases as the first option is more likely to be best. When the first option is no more likely than any other option to be best, subjects make automatic decisions in only 31% of rounds. When the first option is 40% likely to be best, they make automatic decisions in 38% of rounds. When the first option is 50% likely to be the best option, subjects are even more likely to make automatic decisions. At this extreme, they make automatic decisions in 49% of rounds. These levels are significantly different between all treatments at the 1% level using one or two-sided t-tests.

However, there is evidence of switching back and forth between the types of decisions within block. After making the first automatic decision in a block of rounds, there is still a 53.5% chance of making a considered decision in the remainder of the block.

Table 3 also suggests that the frequency of automatic decisions can be influenced by fatigue. In almost all cases, the percentage of inattentive rounds increased as the experiment went along, with the largest jump after the first block of rounds.

D. Estimating DDM Parameters

Krajbich and Rangel (2011) present an extension of DDM to trinary choice and provide...
supporting evidence for their approach. However, how to model trinary (three alternative) choice is still an active debate in the DDM literature (see Ditterich 2010). To estimate the parameters of the DDM, we use the trinary choice model found by Krajbich and Rangel (2011), but neutralize the attentional bias parameter. In this simplified version of their model, there are three integrators (one for each position), all of which start at a value of zero (when \( t = 0 \)): \( E_t^{\text{top}}, E_t^{\text{middle}}, E_t^{\text{bottom}} \). The integration process of each is:

\[
E_t = E_{t+1} + d \times 1_{[\text{best}]} + \epsilon_t,
\]

where \( 1_{[\text{best}]} = 1 \) when the position is best and 0 otherwise. In addition, \( \epsilon_t \) is a draw from a normal distribution with mean 0 and variance \( \sigma^2 \). The relative decision value (RDV) for each position takes this form:

\[
V_t^{\text{top}} = E_t^{\text{top}} - \max\left(E_t^{\text{middle}}, E_t^{\text{bottom}}\right).
\]

When the RDV for a position reaches +1, the item in that position is chosen.

Here we assume each time increment is 1 second (instead of the typically used 1 millisecond). Our empirical strategy is to identify parameters \( d \) and \( \sigma \) that minimize the sum of squared differences between the actual quantile response times and the predicted quantile response times (for the \( .1, .3, .5, .7 \), and \( .9 \) quantiles). To accomplish this, we run 100 simulations for each point on an equally spaced \( 100 \times 100 \) lattice from 0.0001 to 0.1 (for both parameters) and find the point with the minimum sum of squared differences between the quantile response times of the simulations and the quantile response times of considered decisions (16, 32, 52, 78, and 150 seconds). The corresponding parameter values are \( d = 0.0131 \) and \( \sigma^2 = 0.0970 \). To test for goodness-of-fit, we run 10,000 simulations for these parameter values, which produces an average decision quality of 75% correct (as compared to 65% in the actual data).

V. CONCLUDING REMARKS

In this study, we find experimental evidence that is consistent with a dual-process DDM. By adding a fast and frugal strategy as an option for decision makers, we can explain the large number of quick responses we observe, as well as the comparative static predictions with respect to prior beliefs and fatigue.

Our research forms part of a broader movement to expand the data used to fit and to test economic models. Decision time has been the subject of long study within psychology, and with this model, we join two traditions in psychology that use response times to better understand choice. We expect both approaches to have an increasingly important role in economics as response time becomes increasingly studied. In recent years, Wilcox (1993) proposed decision time as a measure of decision costs; Chabris et al. (2009) studied how individuals allocate time across decision problems; Schotter and Trevino (2013) used response time to predict threshold rules; Recalde, Riedl, and Vesterlund (2013) challenged the use of decision time in making inferences about generosity; and Achtziger and Alos-Ferrer (2014) examine the relationship between violations of Bayes’ rule and response times.

Caplin, Dean, and Martin (2011) gather even richer “choice process” data (capturing all provisional choices as well as the final choice and decision time) to test Simon’s model of satisficing. Geng (2015) uses this data to study the role of information in status quo bias. Agranov, Caplin, and Tergiman (2015) use choice process data to show that it takes time for players to reason in the guessing game.

REFERENCES


Ditterich, J. “A Comparison between Mechanisms of Multi-Alternative Perceptual Decision Making: Ability to


SUPPORTING INFORMATION

Additional Supporting Information may be found in the online version of this article: Appendix S1. Experimental instructions