Cognitive Costs and Misperceived Incentives: Evidence from the BDM Mechanism

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Abstract

For the widely-used Becker-DeGroot-Marschak (BDM) mechanism, we provide a Bayesian model of imperfect perception that formalizes the notion of misperceiving incentives and derive population-level comparative static predictions for agents that must pay a cognitive cost to improve their understanding of incentives. These predictions are not symmetric: reductions in mistakes are more robust for cost decreases than for benefit increases. Using data from an existing experiment and new experimental treatments, we find evidence in line with these predictions, suggesting that subject misperceptions respond to both the costs and benefits of better understanding the mechanism’s incentives. Moreover, a treatment that reduces the costs of perception leads to larger improvements in understanding, and these improvements are equivalent to learning with feedback.

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1 Introduction

Strategic dominance is a widespread concept in the theory of incentives and its applications, and mechanisms with dominant strategies are the cornerstone of prominent applications such as matching (deferred acceptance algorithms), auction design (second price auction), belief elicitation (proper scoring rules), and value elicitation (Becker-DeGroot-Marschak mechanism). However, despite the apparent simplicity of the incentives provided by these mechanisms, there is a corpus of evidence showing that individuals systematically fail to play dominant strategies in them (Kagel, Harstad, and Levin 1987; Cason and Plott 2014; Hassidim, Marciano, Romm, and Shorrer 2017; Rees-Jones and Skowronek 2018; Danz, Vesterlund, and Wilson 2020).

Recent experiments from the lab and field have suggested that such failures can arise from difficulties in correctly perceiving the incentives provided by mechanisms. In a school choice setting, Kapor, Neilson, and Zimmerman (2020) find empirical evidence of parents’ systematic misperceptions of match probabilities for an assignment mechanism. In a controlled lab experiment, Danz, Vesterlund, and Wilson (2020) find that for the Binarized Scoring Rule (BSR), a state of the art mechanism for belief elicitation, misreported probabilities exhibit patterns suggestive of biased perception of incentives. In a classroom experiment, Cason and Plott (2014) find that subjects report earnings as if they are facing different incentives than those provided by the Becker-DeGroot-Marschak (BDM) mechanism.

In addition, there is a growing sense that such misperceptions are impacted by the costs and benefits of better understanding the incentives these mechanisms provide. For example, Danz, Vesterlund, and Wilson (2020) note that the BSR has relatively flat incentives at truthful reporting, so that when it comes to misperceptions of incentives, “an exacerbating factor is that the penalties to such misperceptions are small.” When Berry, Fischer, and Guiteras (2020) use the BDM to elicit willingness to pay for clean water technology among farmers, they conclude that “respondents with more at stake may have taken the exercise more seriously.”

In this paper, we offer evidence that misperceptions of the BDM’s incentives respond to both the costs and benefits of better understanding the mechanism’s incentives. As a first step, we provide a Bayesian model of imperfect perception that formalizes the notion of misperceiving incentives, and for a population of agents that must pay a cognitive cost to improve their understanding of incentives, we derive comparative static predictions related to the costs and benefits of more accurately perceiving the mechanism’s incentives. We then investigate these comparative static results experimentally using data from both an existing BDM experiment and new experimental treatments. We find that a treatment that reduces the difficulty of correctly perceiving the BDM’s incentives increases the frequency of dominant strategy play to an extent that is comparable to learning with feedback.
Our model of misperceiving the BDM’s incentives employs standard techniques from the psychology, neuroscience, and economics literatures on perception and attention. We assume that agents receive noisy mental signals about an uncertain state of the world, which is the payoff function they are facing. We follow these literatures in assuming also that agents form correct posterior beliefs about the state based on these signals, and in the context of our model, these beliefs summarize an agent’s uncertainty about the payoff function they are facing.

We use the experimental findings of Cason and Plott (2014) (CP hereafter) to provide an empirically-validated assumption about the form this misperception takes. They observe that subjects sometimes report payments as if they face a first-price sealed-bid procurement auction (FPA hereafter) when they actually face the BDM, so we assume that subjects are uncertain about whether they face the BDM’s payoff function or the FPA’s payoff function. With this assumption about agent confusion, our model generates a one-to-one map between an agent’s beliefs and their offers, which can be used to rationalize many dominated offers. This map can also be used to infer the beliefs agents hold about the likelihood of each payoff function, and with these implied beliefs we measure the extent of misperception subjects possess about the BDM’s incentives.

To provide comparative static predictions for beliefs and offers for this model, we further specialize it by assuming that mental signals are chosen optimally and signals have costs proportional to their informativeness, as in rational inattention theory (Sims 2003). Using this specialized model, we examine the comparative static predictions for a population of rational inattentive agents who may not share the same prior beliefs. We take this approach because our data is generated by a population of individuals who may not share the same initial misperceptions.

We consider population-level comparative statics for two model parameter values: one that represents the benefits of perceiving the mechanism’s incentives better and another that represents the costs of perceiving the mechanism’s incentives better. First, we investigate the comparative static prediction for a model parameter that represents the benefits of perceiving the mechanism’s incentives better: the maximum posted price. As the maximum posted price in the BDM increases, subjects benefit more from understanding the mechanism’s incentives, so our model predicts that their misperception should on average decrease. Thus, their implied beliefs (of the likelihood that they face the BDM’s payoff function) should be higher on average. We are able to test this prediction using data from the CP experiment because they varied the maximum posted price both within subject (across two rounds) and between subjects (within each round).

Using their data, we find that implied beliefs are higher on average when the maximum posted price increases, which is consistent with the theoretical prediction. Despite holding
more accurate beliefs, the average offer is further from the dominant strategy as the maximum posted price increases. However, our theoretical results show that such a divergence is possible as the maximum posted price increases: subjects can make larger mistakes even though their misperception has decreased.¹

The reason for this seemingly counterintuitive result (that some subjects can be made worse off by increasing the benefits of correct perception) is that increasing the benefits of perception through higher stakes not only changes the incentives to understand better, but also, and crucially, it changes the actual game subjects face, and therefore their optimal bidding behavior. And unless subjects perfectly understand the game form, optimal bids get further from the correct bid. In other words, subjects who are moderately mistaken will improve their understanding of the payoff rule on average, but those subjects who are even slightly wrong in their perception are induced by the treatment to make larger mistakes.

The disentangling of these two effects is possible thanks to our Bayesian model, and to the best of our knowledge it is a novel contribution to the understanding of subjects’ mistakes in the BDM mechanism.

Second, we investigate the comparative static prediction for a model parameter that represents the costs of perceiving the mechanism’s incentives better: the cost of information. As the cost of information falls, our model predicts both a decrease in misperception (higher average beliefs of the correct payoff function) and a reduction in the size of mistakes (lower average offers). To test this prediction, we vary the cost of information by comparing a replication of the CP experiment to a new treatment in which the payoffs to each action are specified contingency-by-contingency as in the experimental approach of Esponda and Vespa (2014), Mobius, Niederle, Niehaus, and Rosenblat (2011), and Coffman (2014).²

When we implement this “contingent thinking” protocol for the BDM, we find that the fraction of subjects playing in line with the dominant strategy doubles (an improvement on the extensive margin) and that there is a general shift in offers towards the dominant strategy (an improvement on the intensive margin). Using our model to measure misperception, we find that subjects are 45.6% more correct about the payoff function they face, which corresponds to an average increase of 20.8 percentage points in the likelihood of the correct incentives. Given these results, our comparative static prediction for information costs appears to be consistent with the data as well.

These results suggest a novel form of rational inattention: being rationally inattentive to mechanism incentives. Several other studies have looked for evidence of rational inattention using experiments, but these experiments have instead studied rational inattention to the objects of choice (Caplin, Dean, and Martin 2011; Cheremukhin, Popova, and Tutino 2011;)

¹We say subjects have made a “mistake” if a different offer would have increased their expected payoff.
²See the appendix for the instructions for our replication of the CP experiment and this new treatment.
Martin 2016; Caplin and Martin 2017), in perceptual tasks (Dean and Neligh 2017; Dewan and Neligh 2017; Khaw, Stevens, and Woodford 2017; Ambuehl, Ockenfels, and Stewart 2018), or to opaque taxes (Morrison and Taubinsky 2019). A more closely related paper to ours is by Abeler and Jäger (2015), who consider how work effort changes when tax regimes are more or less complex (in terms of the number of tax rules). They find that subjects who face more complex rules are equally likely to respond to additional tax rules of large and small impact. They note that if agents are certain of the impact of these rules and can only consider a fixed number of rules, then this provides evidence against rational inattention to tax regimes.

Instead, we assume that agents are uncertain about the incentives they face. Our innovation is to model this uncertainty using noisy mental signals. Receiving noisy mental signals is central to signal detection theory, the drift diffusion model, and rational inattention theory. In the economics literature, cognition is modeled with noisy mental signals in Woodford (2014), Caplin and Martin (2015), Caplin and Dean (2015), Matějka and McKay (2015), and Fudenberg, Strack, and Strzalecki (2017). Like many of these papers, we take an as if perspective in which we formally model an otherwise intuitive process.

Also, while it might be appealing from a mathematical perspective, we do not follow a standard approach of assuming that the decision-maker’s mental signal is composed of actual payoffs plus a noise term drawn from some distribution. Instead, we use an empirically-validated assumption about the set of payoff functions that subjects think are possible. In general, collecting non-choice data, such as beliefs about outcomes, can be useful in determining the form that misperception takes. For example, Kapor, Neilson, and Zimmerman (2020) ask subjects about the probability of matches and the details of the matching mechanism, and Martinez-Marquina, Niederle, and Vespa (2019) examine the payoffs mentioned by subjects who are in an advising role. In addition, the set of payoff functions an agent confuses with a given one are potentially testable. For instance, in the appendix we show how the set of optimal actions varies as this set varies, which produces testable content.

Finally, our treatments for varying the cost of better understanding the mechanism’s payoff function draw inspiration from the recent literature on contingent thinking. In fact, difficulties with contingent reasoning have been shown to be a key factor in failures to play dominant strategies (Esponda and Vespa 2017; Li 2017). Since we do not change the extensive form across treatments, we do not ease the cognitive burden on subjects by leveraging obviously strategy-proofness (Li 2017) or resolving uncertainty ex-post or ex-ante (Esponda and Vespa 2017; Martinez-Marquina, Niederle, and Vespa 2019). This is because our goal is to isolate, within the same mechanism, the impact of changing the costs of better perceiving the mechanism’s incentives.

Relatedly, Avoyan and Schotter (2016) use experiments to study the allocation of attention across games when an agent faces more than one game at a time.
1.1 Discussion: Practical Implications

Taken together, our results offer evidence that misperceptions of the BDM’s incentives respond to both the costs and benefits of better understanding the mechanism’s incentives. This suggests that Danz, Vesterlund, and Wilson (2020) are correct to worry that flat incentives in the BSR might lead to noise in probability elicitation and that Berry, Fischer, and Guiteras (2020) are correct in thinking that subjects with more extreme payoffs might come to understand incentives better, leading to variation in the accuracy of preferences elicited by the BDM.

An important follow-up question is to ask whether our results offer insights for practitioners when it comes to the design of experiments. One takeaway from our paper is that when it comes to the BDM mechanism, addressing misperceptions with cost reductions is more robust than with benefit increases.

Our theoretical analysis reveals that whether the average bid improves with an improvement in the benefits of better perception depends on how mistaken a subject initially was. Subjects with an extreme prior belief of being in a FPA would not improve their bidding behavior; on the contrary, it will get further from the correct bid. However, moderately mistaken subjects will strictly improve their bidding behavior on average. Thus, the average effect on the bidding behavior will depend on the distribution of prior beliefs on the population, which cannot be predicted ex-ante.

This has major implications for practitioners, given the intuitive belief that higher stakes alone can suffice to improve strategic choices. This is simply false for the most mistaken part of the population, who can be made worse off with high-powered incentives. Thus, with enough mistaken subjects, the total effect can be negative.

On the other hand, the effect of changing the costs of perceiving the mechanism is more straightforward and perfectly aligns with intuition: lowering the cost of perception improves the average beliefs (Proposition 5) as well the average bidding behavior (Proposition 6). Empirical evidence from our contingent treatment is consistent with these two predictions.

This provides a cautionary tale for practitioners. While high stakes suffice to improve the average understanding of the mechanism, they can magnify strategic mistakes due to misperception, for they impact the optimal strategy and so the total effect is undetermined. Thus, using high stakes to incentivize good understanding of payoff rules may backfire and, while improving understanding, can make a fraction of the population worst off. This is never the case with intervention addressed to improve understanding by lowering costs of perception.

A second takeaway for practitioners is that lowering the costs of perceiving the mechanism’s incentives can be a particularly powerful. Moreover, an important driver of perception
costs appears to be that “thinking through all contingencies is challenging in probabilistic settings” (Martinez-Marquina, Niederle, and Vespa 2019). Martinez-Marquina, Niederle, and Vespa (2019) conjecture that having subjects separately consider payoffs for each contingency makes decisions easier because it focuses a subject’s attention on a subset of random variable realizations, instead of the whole set of potential random variable realizations.

To help subjects think through all contingencies, experimentalists have designed a number of interventions that encourage subjects to consider the payoff implications of their actions contingency-by-contingency. For example, Esponda and Vespa (2017) focus the attention of subjects on a particular contingency by resolving uncertainty. They apply this intervention to five settings and find that subjects are more likely to make the correct decision once a contingency is realized. Martinez-Marquina, Niederle, and Vespa (2019) also aim to help subjects with contingent reasoning by resolving uncertainty, but they instead take an ex-ante perspective by having subjects take a single action that applies across different deterministic states. They alter the acquiring-the-company game by having subjects state a single price at which they would buy a company when both possible companies are available, and they find this intervention substantially reduces choice mistakes.

Another intervention designed to encourage subjects to consider the payoff implications of their actions contingency-by-contingency is one where subjects chose an action in each contingency, a consistent switching point across contingencies is enforced or elicited, and a random choice is selected to be implemented for payment. Healy (2018) calls such an approach the “randomized binary choice” (RBC) elicitation mechanism. Bartling, Engl, and Weber (2015) use an RBC to elicit willingness-to-pay and willingness-to-accept for both money and goods. Mobius, Niederle, Niehaus, and Rosenblat (2011) and Coffman (2014) implement an RBC to elicit probabilities in which subjects chose which “robot” they would let choose for them (by selecting a threshold robot), where each robot had between a 1 and 100 chance of being correct (and a random robot was then selected).

In a development context in the field, Berry, Fischer, and Guiteras (2020) elicit willingness-to-pay for a clean water filter in rural areas of northern Ghana using a Becker-DeGroot-Marschak (BDM) mechanism. After an initial offer is selected, the surveyor shows contingent payoffs in “nearby” contingencies, and then asks if they would like to change their offer.

A common element to these interventions is that they frame information about payoffs contingency-by-contingency. In our paper, we show that this common element is enough on its own to successfully reduce choice mistakes. In doing so, we add to the growing evidence across a wide range settings that the framing of information can be an effective tool for

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4They also use a deterministic belief question to assess whether subjects understand the payoffs for money and find that subjects who appear to understand the payoffs for money still exhibit willingness-to-pay and willingness-to-accept disparities for the good.
making individuals better informed before they make decisions. For instance, frames appear to make it easier for consumers to understand the characteristics of an option (Hastings and Tejeda-Ashton 2008, Choi, Laibson, and Madrian 2009), to navigate the decision problem they face (Abeler and Jäger 2015), or to compare options (Carrera and Villas-Boas 2015, Ericson and Starc 2016, Carpenter, Huet-Vaughn, Matthews, Robbett, Beckett, and Jamison 2019).

2 Model of Misperceiving the BDM’s Incentives

In this section, we first introduce our approach to imperfect perception of the BDM’s incentives, and then solve for optimal offers in the BDM with misperception of its incentives. Finally, we consider how offers and beliefs change with model parameter values if the misperception of agents responds to the costs and benefits of correctly perceiving the BDM’s incentives.

2.1 Misperception and the BDM’s Incentives

The standard BDM is a simple decision problem. A subject who values a good at $\theta$ is asked to choose an “offer” price $b$, which corresponds to the minimum price at which they are willing to sell the good. Next, a “posted” price $p$ is randomly drawn from a uniform distribution on $[0, \bar{p}]$. If $p \geq b$ the good is sold and the agent is paid $p$. Otherwise, the agent keeps the good and obtains $\theta$. It is easy to see that $b_{BDM}^* = \theta$ is a dominant strategy.

In the literature on perception and attention, agents are assumed to receive noisy signals about a state of the world $\omega \in \Omega$. Our innovation is to assume that $\Omega$ is the set of possible payoff functions the agent might confuse with the BDM’s payoff function. While agents start off uncertain about which $\omega \in \Omega$ they face, they are able to reduce their uncertainty by thinking about the payoff function they face. When there are cognitive limits, this can produce game form misrecognition, which is “a failure of the decision maker to recognize the proper connections between the acts available for choice and the consequences of choice” (Cason and Plott 2014).

Based on the empirical evidence in Cason and Plott (2014), we analyze the case where subjects could confuse the BDM’s payoff function with the First Price Auction’s (FPA) payoff function. Thus, the states of the world are $\Omega = \{\omega_0, \omega_1\}$, where $\omega_0$ corresponds to the BDM.

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5For simplicity, in the body of the paper we assume monetary amounts are continuous. In the appendix, we solve for the optimal offer strategy when prices and offers are discrete, and show it converges to the continuous case.

6In the appendix, we show how the optimal strategy changes if we assume instead that subjects confuse
and $\omega_1$ corresponds to the FPA.\footnote{The restriction to a binary state space $\Omega$ is a common assumption in the literature for the sake of analytical tractability. Our model is still in principle solvable with more states.} With the FPA, if the good is sold (i.e. if $p \geq b$) the agent is paid $b$ instead of $p$. Note the action set is identical in both states of the world, as well as the information sets. The only difference is the way an agent believes payoffs are determined.

The prior beliefs that an agent has about the likelihood of facing alternative payoff functions is $\mu \in \Delta(\Omega)$. This prior could be set exogenously by an experimenter, determined by an individual’s experience in a particular laboratory, or formed from day-to-day interactions. Also, it could be correct given the likelihood of encountering a particular set of incentives in a particular setting, but it need not be correct. The prior probability an agent assigns to payoffs being determined by the BDM is given by $\mu(\omega_0)$. For convenience, we abuse notation and denote $\mu := \mu(\omega_0)$ as the prior probability of state of world being $\omega_0$. While determining the prior of an agent about payoff functions is an empirically challenging task, our main comparative statics predictions will be robust to changes in the prior.

Before choosing a strategy, the agent receives a mental signal about the payoff function the face, which represents their subjective perception of the incentives they are facing. This signal can be fully informative about the payoff function, but it can also be fuzzy and stochastic, such as the mental signal received by a subject who does not put much cognitive effort into thinking about the incentives they are facing.\footnote{This differs from Compte and Jehiel (2007), who study the impact of an agent acquiring information about their valuations, not the payoff function itself.}

The agent’s signal is represented by an information structure $\pi : \Omega \rightarrow \Delta(\Delta(\Omega))$, where $\Gamma(\pi) \in \Delta(\Omega)$ is the set of posterior beliefs supported by $\pi$. We restrict information structures to those that contain Bayes plausible posteriors according to the prior $\mu$, which we denote by $\Pi(\mu)$.\footnote{This posterior-based approach is used to model imperfect perception and limited attention by Caplin and Martin (2015), Caplin and Dean (2015), and Matějka and McKay (2015).} For convenience, we abuse notation and denote $\gamma := \gamma(\omega_0)$ as the posterior probability of state of world being $\omega_0$. In other words, $\gamma$ is the agent’s belief of the likelihood they are actually facing the BDM’s incentives when choosing their offer strategy.

The timeline of the decision problem with misperception is summarized in Figure 1. With misperception, the agents may be unsure of which payoff function they face, so their optimal offer strategy will reflect this uncertainty. Based on their posterior belief $\gamma$, the agent chooses a minimum selling price $b$ to maximize the expected payoff accounting for the uncertainty about the payoff generating rule, this is, $\int_0^b \frac{1}{p} \theta dp + \int_b^\bar{p} \frac{1}{p} [\gamma p + (1 - \gamma)b] dp$. The unique optimal offer strategy is given by,

$$b^*(\gamma) = \frac{\theta + (1 - \gamma)\bar{p}}{2 - \gamma}.$$
As long as \( \theta \leq \bar{p} \), the optimal offer is decreasing in \( \gamma \), and for a given \( \gamma \), the optimal offer is increasing in \( \bar{p} \). This optimal offer reduces to the standard optimal strategies when posteriors are degenerate. Specifically, \( b^*(1) = b_{BDM}^* = \theta \) and \( b^*(0) = b_{FPA}^* = \frac{\theta + \bar{p}}{2} \). As a result, all offers in the range \([\theta, \frac{\theta + \bar{p}}{2}]\) can be rationalized by our model.

## 2.2 Offers with Optimal Misperception

Without any additional restrictions on the form of misperception, our model does not indicate how posterior beliefs and offers will change as model parameter values change. A natural restriction on misperception is to assume that information structures \( \pi \) are chosen optimally. We assume further that information carries entropic costs, as in rational inattention theory.

Thus, we assume that after observing \( \theta \), the agent chooses an information structure \( \pi \). Once a posterior \( \gamma \) is observed, an offer price \( b \) is chosen, a posted price \( p \) is realized, and payoffs are determined according to the BDM, regardless of subject’s beliefs.

We solve this model backwards. First, fixing a posterior, the optimal offer strategy is given by (1). Denote the payoff from this strategy by \( u(\gamma) \). Next, considering the cost it will entail, an information structure is chosen optimally, subject to Bayesian plausibility. Specifically, let the cost of any information structure \( \pi \) be

\[
K(\pi, \kappa, \mu) = \kappa \mathbb{E}_\pi [H(\mu) - H(\gamma)]
\]

where \( H(\gamma) \) is the standard Shannon’s entropy. The parameter \( \kappa \) linearly scales the cost of an information structure, and it is typically interpreted it as the marginal cost of information.\(^{10}\)

Given prior \( \mu \), the value of choosing the optimal information structure is

\[
V(\mu) = \max_{\pi \in \Pi(\mu)} \mathbb{E}_\pi [\hat{\nu}(\gamma)] \quad \text{s.t.} \quad \mathbb{E}_\pi [\gamma] = \mu
\]

\(^{10}\)See Sims (2003) and Matějka (2015).
where for all $\gamma \in \Gamma(\pi)$, the function $\hat{v}(\gamma) = u(\gamma) - \kappa(H(\mu) - H(\gamma))$, is the interim net benefit from observing posterior $\gamma$ and acting optimally afterwards.

This problem can be mapped into the sender’s problem in the “costly persuasion” framework of Gentzkow and Kamenica (2014) by choosing the reference belief to be the prior $\mu$ and the value of the induced belief (adjusted for the cost of inducing that belief) to be $\hat{v}(\gamma)$.$^{11}$

Because $V(\mu)$, the value of the above program, is the concave closure of $\hat{v}(\mu)$, the concavity of $\hat{v}$ determines whether or not the agent benefits from choosing an informative information structure or not. We say an information structure is “informative” if it generates posterior beliefs that are different from the prior.

Since the function $\hat{v}$ is twice differentiable on $(0,1)$, it is easily inferred that there are two cases depending on the region of parameters: On the one hand, if $\frac{\kappa \bar{p}}{(\bar{p} - \theta)^2} < \frac{1}{6\sqrt{3}}$, the domain of the function $\hat{v}$ can be partitioned in three regions (strictly concave, strictly convex, and then strictly concave). For the set of priors between the strictly concave regions, $V(\mu) > \hat{v}(\mu)$, so it is optimal to choose an informative information structure. Outside this region, the agent regards the BDM payoff function to be either very unlikely or very likely, so it is not worth choosing an informative, but costly, information structure.

On the other hand, if $\frac{\kappa \bar{p}}{(\bar{p} - \theta)^2} > \frac{1}{6\sqrt{3}}$, $\hat{v}$ is strictly concave for all $\mu$, so it is never optimal to choose an informative information structure. This establishes the following claim.

**Claim 1** There exists a set of priors such that choosing an informative information structure is optimal if and only if $\frac{\kappa \bar{p}}{(\bar{p} - \theta)^2} < \frac{1}{6\sqrt{3}}$.

While an optimal information structure exists for all model parameter values, this condition indicates when the optimal information structure will or will not contain informative signals (in other words, when it is degenerated for all priors). It follows that when this condition holds, the optimal information structure has two posteriors in its support, that we denote by $\gamma^*_1$ and $\gamma^*_2$. Also, these posteriors are unique for a fixed prior. Otherwise, a contradiction with strict concavity would be reached.

It is possible to give a differential characterization of the optimal posteriors, which turns out to be useful for deriving comparative statics. In the spirit of the ILR conditions of Caplin and Dean (2013), these restrictions are:

$^{11}$In the framework of Gentzkow and Kamenica (2014), the reference belief does not need to coincide with the prior. Instead, it can be any fixed interior belief against which the cost of an information structure is assessed. In our case, the reference belief happens to coincide with the prior, which is inconsequential because the optimal information structure is independent of the fixed reference belief.
1. Same slope of $\hat{v}$ at the optimal posteriors:

$$\left. \frac{\partial \hat{v}}{\partial \gamma} \right|_{\gamma^*} = \left. \frac{\partial \hat{v}}{\partial \gamma} \right|_{\gamma^*_2} \quad (2)$$

2. Same tangent of $\hat{v}$ and $V$ at the optimal posteriors:

$$\hat{v}(\gamma^*_1) - \hat{v}(\gamma^*_2) = \gamma^*_1 \left. \frac{\partial \hat{v}}{\partial \gamma} \right|_{\gamma^*_1} - \gamma^*_2 \left. \frac{\partial \hat{v}}{\partial \gamma} \right|_{\gamma^*_2} \quad (3)$$

A geometric intuition for this conditions can be provided using Figure 2. The curved black line corresponds to $\hat{v}$, and the straight red line corresponds to the unique line tangent to $\hat{v}$ at the optimal posteriors. The concave closure of $\hat{v}$ coincides with $\hat{v}$ on the intervals $[0, \gamma^*_1)$ and $(\gamma^*_2, 1]$ and with the tangent line on $[\gamma^*_1, \gamma^*_2]$. The first restriction says that the slope of $\hat{v}$ at the optimal posteriors is the same. The second adds the restriction that the red line has the same slope as $\hat{v}$ at the optimal posteriors.

Finally, if the agent has a prior $\mu \in [0, \gamma^*_1) \cup (\gamma^*_2, 1]$, the prior and posterior will coincide, so the agent’s optimal offer is $b^*(\mu)$. Otherwise, if the agent’s prior belongs to $[\gamma^*_1, \gamma^*_2]$, the posterior is stochastically determined to be one of the two optimal posteriors characterized above, so the agent’s offer will sometimes be $b^*(\gamma^*_1)$ and sometimes be $b^*(\gamma^*_2)$.
2.3 Comparative Statics of Optimal Misperception

In this subsection we establish comparative static predictions for two parameters of our specialized model: the maximum posted price $\bar{p}$ and the cost of information $\kappa$. We first examine how a change in these two parameters impacts the optimal posteriors (Claims 2 and 3). Building on these results, we derive comparative statics for average optimal posteriors, where the average is with respect to the optimal information structure $\pi$ (Propositions 4 and 5). Finally, Subsection 2.3.2 summarizes our findings for how the average offer responds to changes in $\bar{p}$ and $\kappa$.

2.3.1 Comparative Statics: Optimal Posteriors

The maximum posted price ($\bar{p}$) and the cost of information ($\kappa$) impact optimal posteriors in systematic ways. As $\bar{p}$ increases, an agent has more incentive to obtain informative mental signals of the payoff function, so the region of priors for which it is optimal to do so enlarges. On the other hand, as $\kappa$ increases, the region of priors for which it is optimal to obtain informative mental signals of the payoff function shrinks. These comparative statics are established with the following two claims.

Claim 2 Assume $\frac{\kappa \bar{p}}{(\bar{p} - \theta)^2} < \frac{1}{6\sqrt{3}}$ and without loss of generality that $\gamma_1^* < \gamma_2^*$. Optimal posterior $\gamma_1^*$ decreases with $\bar{p}$, and optimal posterior $\gamma_2^*$ increases with $\bar{p}$.

Proof. Use the implicit function theorem to derive conditions (2) and (3) implicitly with respect to $\bar{p} > 0$ and solve for $\hat{v}'(\gamma_i) \frac{\partial \gamma_i}{\partial \bar{p}}$, $i = 1, 2$ to obtain

$$\frac{\bar{p} - \theta}{\gamma_j - \gamma_i} \frac{\partial \gamma_j}{\partial \bar{p}} = \frac{1}{\gamma_j - \gamma_i} \frac{\gamma_j - \gamma_i}{2(2 - \gamma_j)} \left(\frac{\gamma_j - \gamma_i}{2(2 - \gamma_j)}\right)^2 \frac{\gamma_j - \gamma_i}{2(2 - \gamma_j)^2 \bar{p}^2}$$

$i \neq j, i, j = 1, 2$. Since $\gamma_1 < \gamma_2$, the right-hand term is positive for $i = 1$ and negative for $i = 2$. Combined with the fact that $\hat{v}$ is strictly concave in the optimal posteriors, this implies $\frac{\partial \gamma_1}{\partial \bar{p}} < 0$ and $\frac{\partial \gamma_2}{\partial \bar{p}} > 0$, as desired.

Claim 3 Assume $\frac{\kappa \bar{p}}{(\bar{p} - \theta)^2} < \frac{1}{6\sqrt{3}}$ and without loss of generality that $\gamma_1^* < \gamma_2^*$. Optimal posterior $\gamma_1^*$ increases with $\kappa$, and optimal posterior $\gamma_2^*$ decreases with $\kappa$.

Proof. Since by assumption $\gamma_1^* < \gamma_2^*$, use again the implicit function theorem to derive conditions (2) and (3) implicitly with respect to $\kappa$, and solving for $\hat{v}'(\gamma_i) \frac{\partial \gamma_i}{\partial \kappa}$ for $i = 1, 2$, we obtain:

$$\frac{\bar{p} - \theta}{\gamma_1 - \gamma_2} \frac{\partial \gamma_1}{\partial \kappa} = \frac{H(\gamma_1) - H(\gamma_2) - (\gamma_1 - \gamma_2)H'(\gamma_i)}{\gamma_1 - \gamma_2}$$

13
Since $\gamma_1 < \gamma_2$ by assumption, the denominator in both expressions is negative. It follows from strict concavity and differentiability of the entropic cost function that $H(\gamma_1) - H(\gamma_2) + H'(\gamma_1)(\gamma_2 - \gamma_1) \geq 0$ and $H(\gamma_1) - H(\gamma_2) - H'(\gamma_2)(\gamma_1 - \gamma_2) \leq 0$. Therefore, the right-hand side is negative for $i = 1$, and is positive for $i = 2$. Since $\hat{v}$ is concave in the optimal posteriors, we have that $\frac{\partial H}{\partial \gamma_i} \geq 0$ and $\frac{\partial H}{\partial \gamma_2} \leq 0$, as desired. ■

Building on previous claims, we first show that when agents face the BDM, the average belief is increasing in the maximum posted price ($\bar{p}$).\(^{12}\) Intuitively, as the benefits to perceiving the payoff function correctly increase, so do the average beliefs of the correct payoff function.

**Proposition 4** Let $E[\gamma|\omega_0, \bar{p}]$ be the expected belief when facing the BDM and the maximum posted price is $\bar{p}$. If $\bar{p}' > \bar{p}$, then $E[\gamma|\omega_0, \bar{p}'] \geq E[\gamma|\omega_0, \bar{p}]$ for any prior $\mu$.

**Proof.** By definition,

$$E[\gamma|\omega_0] = \pi(\gamma_1^*|\omega_0)\gamma_1^* + \pi(\gamma_2^*|\omega_0)\gamma_2^* = \frac{\pi \gamma_1^*}{\mu} \gamma_1^* + \frac{(1 - \pi) \gamma_2^*}{\mu} \gamma_2^*$$

with $\gamma_1^*$, $\gamma_2^*$, and $\pi$ implicit functions of $\bar{p}$. By definition $\pi$ is constrained to satisfy

$$\gamma_1^* \pi + \gamma_2^*(1 - \pi) = \mu$$

so we can rewrite

$$E[\gamma|\omega_0] = \frac{1}{\mu} \left( \frac{\gamma_2^* - \mu}{\gamma_2^* - \gamma_1^*} \right) (\gamma_1^*)^2 + \frac{1}{\mu} \left( 1 - \frac{\gamma_2^* - \mu}{\gamma_2^* - \gamma_1^*} \right) (\gamma_2^*)^2 = \gamma_2^* + \gamma_1^* - \frac{\gamma_2^* \gamma_1^*}{\mu} \tag{4}$$

By using the chain rule to derive $E[\gamma|\omega_0]$ w.r.t. $\bar{p}$, we obtain

$$\frac{dE[\gamma|\omega_0]}{d\bar{p}} = \frac{\partial E[\gamma|\omega_0]}{\partial \gamma_1^*} \frac{\partial \gamma_1^*}{d\bar{p}} + \frac{\partial E[\gamma|\omega_0]}{\partial \gamma_2^*} \frac{\partial \gamma_2^*}{d\bar{p}}$$

and because $\frac{\partial E[\gamma|\omega_0]}{\partial \gamma_1^*} < 0$ and $\frac{\partial E[\gamma|\omega_0]}{\partial \gamma_2^*} > 0$ (from 4), we conclude from Claim 2 that $\frac{dE[\gamma|\omega_0]}{d\bar{p}} > 0$ as desired. ■

Moreover, when agents face the BDM, the average belief decreases as the cost of information ($\kappa$) increases. As it gets harder and harder to disentangle states, the extent of misperception increases on average.

**Proposition 5** Let $E[\gamma|\omega_0, \kappa]$ be the expected belief when facing the BDM and the cost of information is $\kappa$. If $\kappa' > \kappa$, then $E[\gamma|\omega_0, \kappa'] \leq E[\gamma|\omega_0, \kappa]$ for any prior $\mu$.

\(^{12}\) We condition the state on when agents face the BDM because our interest is in the comparative statics of optimal misperception when subjects are facing the BDM.
Proof. Using the chain rule to derive $E[\gamma|\omega_0]$ w.r.t. $\kappa$, we obtain
\[
\frac{dE[\gamma|\omega_0]}{d\kappa} = \frac{\partial E[\gamma|\omega_0]}{\partial \gamma_1^*} \frac{\partial \gamma_1^*}{\partial \kappa} + \frac{\partial E[\gamma|\omega_0]}{\partial \gamma_2^*} \frac{\partial \gamma_2^*}{\partial \kappa}
\]
and because $\frac{\partial E[\gamma|\omega_0]}{\partial \gamma_1^*} < 0$ and $\frac{\partial E[\gamma|\omega_0]}{\partial \gamma_2^*} > 0$ (see eq. 4), from Claim 3 we conclude that $\frac{dE[\gamma|\omega_0]}{d\kappa} \leq 0$, as desired. \[\square\]

2.3.2 Comparative Statics: Average Offers

Last, we examine how average offers change as the maximum posted price ($\bar{p}$) and the cost of information ($\kappa$) change. When agents are facing the BDM, the average offer can either increase or decrease with $\bar{p}$ depending on the prior, but the average offer increases with $\kappa$ for all priors.

Because the average optimal posterior increases with $\bar{p}$, we might expect the average offer to decrease with $\bar{p}$. However, this is not true in general, as the example in Figure 3 illustrates. Note that in general, the expression for the average offer, conditional on facing the BDM, is given by
\[
E[b(\gamma)|\omega_0, \mu] = \begin{cases} 
\left[\gamma_2^* A - \gamma_1^* B\right]_{\mu} - (A - B) & \text{if } \mu \in [\gamma_1^*, \gamma_2^*], \\
b(\mu) & \text{otherwise}.
\end{cases}
\]
where $A = \frac{\gamma_2^*}{\gamma_2^* - \gamma_1^*} b(\gamma_1^*)$ and $B = \frac{\gamma_2^*}{\gamma_2^* - \gamma_1^*} b(\gamma_2^*)$. Observe that the expected offer is convex in the region of the domain where information is valuable.

When it is not valuable to get informative mental signals, $b^*(\gamma)$ shifts out as $\bar{p}$ increases, which means that there is an increase in the expected offer for all such $\mu$. At the same time, an increase in $\bar{p}$ widens the region of priors for which it is optimal to get informative mental signals. For such priors, when $\bar{p}$ increases, the rate at which the average offer moves closer to the standard one increases. The net result is that the comparative static is uncertain and any behavior can be rationalized by assuming a particular prior (or a particular distribution of priors across subjects).

On the other hand, when $\kappa$ increases, the average offer moves in a consistent direction for all priors. A higher information cost decreases the value of disentangling the two states, and as a result, the agent misperceives the payoff function more often, resulting in a higher average offer. This is illustrated in Figure 4 and formally established in the following proposition.

Proposition 6 Let $E[b^*(\gamma)|\omega_0, \kappa]$ be the expected offer when facing the BDM and the cost of information is $\kappa$. For $\kappa' > \kappa$, then $E[b^*(\gamma)|\omega_0, \kappa'] \geq E[b^*(\gamma)|\omega_0, \kappa]$ for any prior $\mu$. 


Figure 3: An example of $E[b^*(\gamma)|\omega_0]$ where $\bar{p} = 4$ (blue), $\bar{p} = 5$ (black) $\bar{p} = 6$ (red), and $\kappa = 0.1$.

Figure 4: An example of the impact of an increase in the cost of information on the average expected offer when agents are facing the BDM. The dashed line corresponds to a higher $\kappa$ than the one for the solid line.
Proof. For convenience, let \( g(\mu, \kappa) = E[b^*(\gamma)|\omega_0] \). First, note that by Claim 1, for \( \kappa' \geq \frac{\theta^2}{6\sqrt{bp}} \), \( g(\mu, \kappa') = b^*(\mu) \) as choosing an uninformative information structure is optimal for every prior and the claim is trivially satisfied. For \( \kappa < \frac{\theta^2}{6\sqrt{bp}} \), let \( \gamma^*_i(\kappa) \leq \gamma^*_2(\kappa) \) be the unique posteriors in the optimal signal structure for any \( \mu \in [\gamma^*_1(\kappa), \gamma^*_2(\kappa)] \). Claim 3 implies that for any \( \kappa' > \kappa \), \( [\gamma^*_1(\kappa'), \gamma^*_2(\kappa')] \subseteq [\gamma^*_1(\kappa), \gamma^*_2(\kappa)] \). Therefore, we have three cases: In the first case, \( \mu \in [0, \gamma^*_1(\kappa)] \cup [\gamma^*_2(\kappa), 1] \), so we have \( g(\mu, \kappa) = b^*(\mu) = g(\mu, \kappa') \).

In the second case, \( \mu \in [\gamma^*_1(\kappa), \gamma^*_1(\kappa')] \cup [\gamma^*_2(\kappa'), \gamma^*_2(\kappa)] \), we have \( g(\mu, \kappa') = b^*(\mu) \), and since \( b^*(\mu) \) is strictly concave in \( \mu \) we have \( g(\mu, \kappa') > \pi b^*(\gamma^*_1(\kappa')) + (1 - \pi) b^*(\gamma^*_2(\kappa')) \) for \( \pi \) s.t. \( \pi \gamma^*_1(\kappa) + (1 - \pi) \gamma^*_2(\kappa) = \mu \). Since \( b^*(\cdot) \) is decreasing, and \( \frac{\pi \gamma^*_1(\kappa')}{\mu} < \pi, \frac{(1-\pi)\gamma^*_2(\kappa')}{\mu} > 1 - \pi \), we have that \( \pi b^*(\gamma^*_1(\kappa)) + (1 - \pi) b^*(\gamma^*_2(\kappa)) > \frac{\pi \gamma^*_1(\kappa')}{\mu} b^*(\gamma^*_1(\kappa)) + \frac{(1-\pi)\gamma^*_2(\kappa')}{\mu} b^*(\gamma^*_2(\kappa)) = g(\mu, \kappa) \), so the conclusion follows.

In the third case, \( \mu \in [\gamma^*_1(\kappa'), \gamma^*_2(\kappa')] \), with \( g(\mu, \kappa') = \frac{\pi \gamma^*_1(\kappa')}{\mu} b^*(\gamma^*_1(\kappa')) + \frac{(1-\pi)\gamma^*_2(\kappa')}{\mu} b^*(\gamma^*_2(\kappa')) \) for \( \pi' \) s.t. \( \pi' \gamma^*_1(\kappa') + (1 - \pi') \gamma^*_2(\kappa) = \mu \). Since \( g(\mu, \kappa) \) can be written as (see next paragraph) \( \pi' g(\gamma^*_1(\kappa'), \kappa) + (1 - \pi') g(\gamma^*_2(\kappa'), \kappa) \), from strict concavity of \( b \) and the definition of \( g(\mu, \kappa') \) we conclude the desired inequality.

To see that \( g(\mu, \kappa) = \pi' g(\gamma^*_1(\kappa'), \kappa) + (1 - \pi') g(\gamma^*_2(\kappa'), \kappa) \), note that \( g(\mu, \kappa), g(\gamma^*_1(\kappa'), \kappa), \) and \( g(\gamma^*_2(\kappa'), \kappa) \) are by definition convex combinations of \( b^*(\gamma^*_i(\kappa)) \), \( i = 1, 2 \) with the distribution over \( \gamma^*_i(\kappa) \) such that they average average to the corresponding prior, i.e. \( \pi' \gamma^*_1(\kappa') + (1 - \pi') \gamma^*_2(\kappa) = \mu, \pi' \gamma^*_1(\kappa') \gamma^*_1(\kappa) + (1 - \pi') \gamma^*_2(\kappa) = \gamma^*_1(\kappa), \pi' \gamma^*_1(\kappa') \gamma^*_2(\kappa) + (1 - \pi') \gamma^*_2(\kappa) = \gamma^*_2(\kappa') \). These necessary conditions combined with the fact that \( \pi' \gamma^*_1(\kappa') + (1 - \pi') \gamma^*_2(\kappa') = \mu \) leads to what is required for the equality of interest to hold.

3 Using Existing Data to Study Comparative Statics

In this section, we test our comparative static prediction for benefits by re-examining existing data. In the BDM experiment of CP, the benefits for accurate perception are varied because the maximum posted price varies, which allows us to examine if the level of misperception changes when the benefits to correctly perceiving the payoff function change. Looking across maximum posted prices, we find evidence that misperception decreases when there are higher benefits to correctly perceive the payoff function, which is consistent with the theoretical prediction.

3.1 Offers in the CP Experiment

In the CP experiment, subjects were provided with a physical card that gave instructions for a seller version of a sealed-bid BDM mechanism. CP used the BDM to elicit the amount
Table 1: Regression Analysis of the CP Experiment. This table presents coefficients of linear regressions where the dependent variable is (1) the offer price for the card, (2) a dummy variable indicating whether the offer price is “near $2” (within $0.05 of $2), or (3) the implied belief of consistent subjects. Robust standard errors are clustered at the subject level.

<table>
<thead>
<tr>
<th></th>
<th>(1) Offer Price</th>
<th>(2) Near $2</th>
<th>(3) Implied Belief</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum Posted Price</td>
<td>0.216***</td>
<td>0.004</td>
<td>0.032**</td>
</tr>
<tr>
<td></td>
<td>(0.040)</td>
<td>(0.013)</td>
<td>(0.014)</td>
</tr>
<tr>
<td>Round</td>
<td>-0.257***</td>
<td>0.144***</td>
<td>0.141***</td>
</tr>
<tr>
<td></td>
<td>(0.086)</td>
<td>(0.026)</td>
<td>(0.029)</td>
</tr>
<tr>
<td>Constant</td>
<td>2.213***</td>
<td>0.002</td>
<td>0.178*</td>
</tr>
<tr>
<td></td>
<td>(0.257)</td>
<td>(0.081)</td>
<td>(0.104)</td>
</tr>
<tr>
<td>Observations</td>
<td>489</td>
<td>489</td>
<td>380</td>
</tr>
</tbody>
</table>

Note: Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

of money subjects would be willing to accept for the card itself, which could be exchanged later for $2 if not sold. Thus, if the payoff function was perceived perfectly, the dominant strategy was for subjects to offer $2 for their card.

After receiving the card, subjects provided an offer price for their card and then turned over the card to reveal a posted price and filled in their actual payments in light of this posted price. In a second round, subjects were then given a new card, also worth $2, and completed the BDM again.

The minimum posted price was always $0, but between subjects, the maximum draw for the posted price $\bar{p}$ varied between $4$, $5$, $6$, $7$, and $8$, and within subject, the maximum posted price $\bar{p}$ varied in the same way between cards.

As described in detail in the preceding section, our model predicts that offers will increase for some parameter values and decrease for others. CP report that the mean offer in their experiment is mostly increasing with the maximum draw $\bar{p}$, and for offers outside of $0.05$ of $2$, the mean offer is monotonically increasing in the value of the maximum draw $\bar{p}$. Table 1 provides the estimates from a linear regression of offers onto the maximum posted price and round with robust standard errors clustered at the subject level. In this regression, the coefficient on maximum posted price is positive and statistically significant (coefficient=0.216, p-value<0.001). This means that a $1$ increase in the maximum posted price corresponds to an approximately $0.22$ increase in offers.
While average offers differ between maximum posted prices, the percent of subjects who offered within $0.05 of $2 does not appear to increase with the value of the maximum posted price $\bar{p}$. As shown in Table 1, for a linear regression of a dummy variable indicating whether an offer is “near $2$” (within $0.05$ of $2$) onto the maximum posted price and round, the coefficient on maximum posted price is not statistically significant (coefficient=0.004, p-value=0.783 with robust standard errors clustered at the subject level). This suggests that changes in the benefits to accurate perception largely impact the intensive margin of perception, which helps to justify models of partial misperception.

3.2 Using Implied Beliefs to Measure Misperception

Assuming subjects misperceive the payoff function, we can use their offers to determine their “implied” beliefs of how likely they are to be facing the BDM by inverting the optimal offer function (1). Specifically,

$$\gamma = \frac{\theta + \bar{p} - 2b}{\bar{p} - b}$$

Because $\bar{p}$ is in this equation, the same offer implies different beliefs depending on the range of posted prices.\(^{13}\) The map between implied beliefs and offers for the different posted price ranges used in the CP experiment is illustrated in Figure 5. The fact that the same offer corresponds to different implied beliefs for different maximum posted prices will play a central role in our subsequent analysis because when looking across posted price ranges, increases in offers (moving away from the dominant strategy) may not correspond to increases in misperception about the mechanism’s payoff function.

Because beliefs are bounded between 0 and 1, this relationship places restrictions on the offers that are consistent with our model.\(^{14}\) Across posted price ranges, 75.9% of offers are consistent with our model for the first card and 79.5% are consistent with our model for the second card.\(^{15}\) These rates are not statistically different (two-sided p-value=0.3402 using a test of proportions), which means that we do not have evidence that experience increases consistency with our model.

\(^{13}\)Because increments of the posted price are small ($0.01$), the discretized version of this equation is virtually indistinguishable. In practice, the difference in the continuous and discrete versions is less than half a percentage point, so we use the continuous version throughout our analysis.

\(^{14}\)We allow a 5 percentage point margin, so implied beliefs between -0.05% and 1.05% sure of the correct payoff function are considered consistent with our model.

\(^{15}\)If an agent were to choose offers randomly between $0$ and $\bar{p}$, then approximately 25% of offers would be consistent with our model when $\bar{p} = 4$, and approximately 38% would be consistent when $\bar{p} = 8$. 

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3.3 Comparative Statics for Benefits

As we have shown in our theoretical comparative static analysis, the average belief of the BDM should increase with $\bar{p}$. In other words, misperception should fall as benefits to accurate perception increase. We now use the implied beliefs of subjects in the CP experiment to test this prediction.

We find a clear trend in implied beliefs for consistent subjects. As shown in Table 1, a linear regression of offers onto the maximum posted price and round for consistent subjects yields a coefficient on maximum posted price that is positive and statistically significant (coefficient=0.032, p-value=0.022 with robust standard errors clustered at the subject level). This means that holding the round fixed, increasing the maximum posted price by $1 corresponds to an approximately 3 percentage point increase in beliefs of the correct payoff function.

These results provide a key insight about mistakes in the BDM. Because choice mistakes are increasing in size with $\bar{p}$, it might seem that subjects are becoming less informed about the mechanism. However, our regression results suggest that misperception is actually decreasing with the maximum posted price.

As established previously, a higher maximum posted price should lead subjects to be more certain of payoff function, but the impact of changing the maximum posted price on offers is ambiguous. Using the map between beliefs and offers, we can determine whether both mistakes and misperception are increasing or whether mistakes are increasing despite a
decrease in misperception.

4 Using New Data to Study Comparative Statics

In this section, we test our comparative static prediction for costs by running new experiments. To vary the cost of information, we use variation in the experimental protocol. Specifically, we implement both a replication of the CP experiment and a new protocol in which payoffs are explained contingency-by-contingency.

4.1 Our Replication of the CP Experiment

We first replace the CP experiment with a new subject pool and setting. Instead of undergraduate students completing the experiment in a classroom as in CP, our participants were all part of the Kellogg School of Management panel on Amazon’s Mechanical Turk (MTurk) and completed our experiment online.\textsuperscript{16} Subjects in this panel are required to be U.S. residents over the age of 18. Following Cavallo, Cruces, and Perez-Truglia (2017), we used many of the best practices identified in the literature for getting high quality responses when running studies on MTurk.\textsuperscript{17} Even with these methods, we received some offers well over the maximum posted price $\bar{p}$. These outliers could represent data entry errors and are not consistent with any of the explanations we considered, so we did not include offers over the maximum posted price in any of the analyses in this section. For our baseline replication, this reduced our sample by 3.6%.\textsuperscript{18} After removing these offers, our analysis sample for the baseline replication consisted of 190 subjects (57.4% Female; $M_{age} = 38.9$, $SD_{age} = 12.6$).

Because our subjects took our experiment online, we needed to make one substantial change to the design, which was to replace the card with a digital token.\textsuperscript{19} We made two other changes. First, we made the value of the token $1$ instead of $2$. Second, we told subjects the distribution from which posted prices were drawn. In our baseline replication, we chose to draw posted prices uniformly from $0$ to $3$ in increments of $0.50$. In practice, it is challenging to implement the BDM at finer increments because payments become cumbersome, but as a robustness check, we also run a version with increments of $0.01$.

Despite differences in pools, settings, and design, we find that the baseline replication

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\textsuperscript{16}Bull, Courty, Doyon, and Rondeau (2019) also replicate the CP experiment, but with university students and in a classroom as in CP.

\textsuperscript{17}This included requiring subjects to complete a standard attentional check question at the start of the experiment, which was passed by 97.0% of subjects who started the baseline replication.

\textsuperscript{18}For technical reasons, CP also removed offers over $\bar{p}$ in their maximum likelihood estimation procedure, but this was just 0.8% of second round offers.

\textsuperscript{19}See the appendix for the instructions for this replication.
Table 2: Summary statistics for our replications and rescaled offers from the CP experiment. Offers in the CP experiment are divided in half. An offer is “near $1” if it is within $0.05 of $1. Standard errors are in parentheses. p-values are from two-tailed tests of proportion for percents and from two-tailed Wilcoxon rank-sum tests otherwise. *** p<0.01; ** p<0.05; * p<0.10.

<table>
<thead>
<tr>
<th></th>
<th>Baseline replication</th>
<th>CP round 1 (rescaled)</th>
<th>Baseline=CP p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observations</td>
<td>190</td>
<td>245</td>
<td></td>
</tr>
<tr>
<td>Mean offer</td>
<td>$1.70 ($0.73)</td>
<td>$1.63 ($0.56)</td>
<td>0.3764</td>
</tr>
<tr>
<td>Percent offer near $1</td>
<td>17.4%</td>
<td>17.6%</td>
<td>0.9603</td>
</tr>
<tr>
<td>Mean offer if not near $1</td>
<td>$1.85 ($0.72)</td>
<td>$1.76 ($0.53)</td>
<td>0.0844*</td>
</tr>
</tbody>
</table>

(a) Baseline replication v. CP round 1.

<table>
<thead>
<tr>
<th></th>
<th>Baseline replication</th>
<th>Robustness check</th>
<th>Baseline=Robustness p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observations</td>
<td>190</td>
<td>92</td>
<td></td>
</tr>
<tr>
<td>Mean offer</td>
<td>$1.70 ($0.73)</td>
<td>$1.58 ($0.64)</td>
<td>0.1792</td>
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<tr>
<td>Percent offer near $1</td>
<td>17.4%</td>
<td>18.5%</td>
<td>0.8190</td>
</tr>
<tr>
<td>Mean offer if not near $1</td>
<td>$1.85 ($0.72)</td>
<td>$1.71 ($0.64)</td>
<td>0.0845*</td>
</tr>
</tbody>
</table>

(b) Baseline replication v. robustness check.
As shown in Table 2, the proportion of offers near $1 is similar (17.6 for rescaled CP offers across posted prices ranges and 17.4 for our replication) and not statistically different using a two-tailed test of proportions (p-value=0.9603). The mean of the rescaled CP offers across posted price ranges in the first round (1.70) is also similar to the mean of the offers in our baseline replication (1.63), and the distributions are not statistically different using a two-tailed Wilcoxon rank-sum test (p-value=0.3764). Figure 6 provides these two distributions side-by-side.

As mentioned previously, we also tested the robustness of our replication by having 92 subjects complete our replication but with posted prices drawn from $0.01 increments. We find more offers between $0.50 increments when posted prices are drawn from smaller increments, but as shown in Table 2, the percent of subjects offering near $1 is similar (17.4% for the baseline replication and 18.5% for the robustness check) and the distribution are not statistically different using a two-tailed Wilcoxon rank-sum test (p-value=0.1792).\textsuperscript{20}

In practice, it is common to tell subjects the dominant strategy of a mechanism, with the hope of avoiding problems of misperception entirely. With this in mind, we ran an additional

\textsuperscript{20}We also ran a robustness check in which 137 subjects first completed our robustness check and then either our baseline replication or our contingent protocol, and those results appear in the appendix.
robustness check in which we informed subjects about the dominant strategy in the area just above the box where they entered their offers. Specifically, we stated: “The rule for selling the token is given below. It is a bit unusual, but its implications are straightforward. There is no way of gaming the rule, the BEST thing that you can do is to ask yourself how much you would be willing to exchange the token for, and then offer the number closest to that amount.” For the 44 subjects who saw this statement, the average offer price ($1.58) and the percent offering near $1 (18.2%) were very similar to the corresponding figures for the robustness check without this statement.\footnote{For this additional robustness check, we also used $0.01 increments in the distribution of posted prices.}

One reason why it might not be effective to tell the subjects the dominant strategy is if subjects do not trust this guidance. In a post-experiment questionnaire, we asked subjects: “Did you trust the statements ‘there is no way of gaming the rule’ and ‘the BEST thing that you can do is to ask yourself how much you would be willing to exchange the token for’?” While 81.8% of subjects indicated that they trusted the statement, the average offer price was higher among such subjects ($1.94), though the difference is not statistically significant (two-sided p-value = 0.6451 using a test of proportions). Also, subjects who trust the statements might not bother to read the payoff rule listed at the bottom of the screen if doing so is costly, so we also asked subjects who answered yes to the previous question: “Did you decide to read the rule anyway?” 97.2% of these subjects indicated that they decided to read the rule despite trusting it. If subjects are attempting to read the rule anyway, then it is possible that even in this treatment, subjects are misperceiving the mechanism and acting based on their fuzzy perception of the mechanism’s payoff function.

\section{4.2 A New Contingent Protocol}

To test the comparative static prediction, our goal is to make it “easier” for a subject to understand the BDM’s payoff function. However, because our model does not provide an explicit map between experimental protocols and the cost of information, we draw inspiration from the literature, primarily the method of explaining payoffs contingency-by-contingency, as in Esponda and Vespa (2014).

In our “contingent” protocol, we frame each posted price as a separate computer bidder, one of which the subject will be paired with. Each bidder offers a different bid, and like the posted prices in the baseline replication, they are spaced in $0.50 increments. The payoff function is identical to the one in the baseline replication: if the offer (the “minimum amount you are willing to sell the token for”) is at or below the computer’s bid, the subject sells the token at the computer’s bid.\footnote{The exact instructions are provided in the appendix.}
This protocol has similarities to methods for eliciting valuations that have been proposed in the experimental literature. Mobius, Niederle, Niehaus, and Rosenblat (2011) and Coffman (2014) elicit probabilities that has a very similar framing to ours. In their task, subjects chose which robots they would let choose for them, where each robot had between a 1 and 100 chance of being correct. Healy (2018) outlines a “randomized binary choice” (RBC) elicitation mechanism in which subjects chose at each price whether they would like to sell at that price and a consistent switching point is enforced. For payment, a random choice is selected to be implemented.23

4.3 Comparative Statics for Costs

Our analysis sample for the contingent protocol consisted of 192 subjects (59.4% Female; \(M_{\text{age}} = 39.3, SD_{\text{age}} = 12.9\)). As shown in Table 3, the average offer from these subjects was $1.50, which is $0.20 lower than the average offer from subjects in the baseline replication. The distribution of offers for the contingent protocol and the baseline replication are provided in Figure 7, and these distributions are significantly different at a 5% level (\(p=0.0016\) for a two-tailed Wilcoxon rank-sum test). Importantly, 34.9% of offers were near $1 with the contingent protocol, which is roughly double the percentage in the baseline replication, and these proportions are significantly different at a 5% level (\(p=0.0001\) for a two-tailed test of proportions). In addition, there is a much higher average implied belief for the contingent protocol. The average implied belief for consistent subjects increases by over 20 percentage points, and the average implied belief is significantly different at a 5% level (\(p<0.0001\) for a two-tailed Wilcoxon rank-sum test). Both of these findings – the decrease in average offers and the increase in average implied beliefs – are consistent with the theoretical comparative static prediction for the cost of information.

We also find that our contingent protocol produced a similar distribution of offers to the second round of CP experiment, at which point subjects had a round of experience with the BDM and had received feedback by calculating their first round payment. The proportion of offers near $1 is similar (34.9% of offers with our contingent protocol and 31.0% of rescaled second round offers in the CP experiment) and not statistically different using a two-tailed test of proportions (\(p=0.3915\)). The mean of the offers ($1.50) is also similar to the mean of the rescaled second round offers in the CP experiment ($1.49), and the distributions are not statistically different using a two-tailed Wilcoxon rank-sum test (\(p=0.5933\)).

\footnote{Bartling, Engl, and Weber (2015) use a “multiple price list” RBC to elicit willingness-to-pay (WTP) and willingness-to-accept (WTA) for both money (as in CP) and goods. They also use a deterministic belief question to assess whether subjects understand the payoffs for money and find that subjects who appear to understand the payoffs for money still exhibit WTP-WTA disparities for the good. Brebner and Sommers (2018) compare choices between the BDM and the multiple price list RBC and find that the WTP-WTA gap is similar between for the protocols they use.}
Figure 7: Offers in our baseline replication and new protocol.

Table 3: Summary statistics for our new protocol and baseline replication. An offer is “near $1” if it is within $0.05 of $1. Standard errors are in parentheses. p-values are from two-tailed tests of proportion for percents and from two-tailed Wilcoxon rank-sum tests otherwise. *** p<0.01; ** p<0.05; * p<0.10.
4.4 Other Explanations for Mistaken Offers

Because there are many possible explanations for mistakes in BDM experiments besides misperception, we use this section to investigate several of these possibilities. We examine not just their ability to explain the mistakes observed in the CP experiment and our experiments, but their ability to explain the difference in offers between our baseline replication and contingent protocol.

4.4.1 Other Behavioral Biases

A number of behavioral biases could impact the offers made in BDM experiments.\footnote{\textcite{CP and Brebner and Sonnemans (2018)} provide a comprehensive and detailed discussion of the possible explanations for mistakes in the BDM.} For example, the endowment effect leads to higher offers in the BDM because ownership creates a reference point from which losses are experienced, and the direct benefits to ownership could lead to higher offers in the BDM because ownership creates positive feelings that make the good feel higher valued. With bad deal aversion, agents could set higher offers in the BDM as not to get a bad deal relative to a reference point when selling the good, and with...
the buy-low sell-high heuristic, they might follow an optimal rule from the world in which first offers are higher (in anticipation of future bargaining).

However, many of these behavioral theories do not predict a change in offers between our baseline replication of the CP BDM and our new protocol because we hold the extensive form, payoffs, and role fixed between the two protocols. For instance, behavioral biases based on ownership of the token, such as the endowment effect and the direct benefits of ownership such as positive feelings about the good, do not suggest a change in behavior, given that ownership is unaffected by our change in protocol. In addition, behavioral biases based on the maximum payment, such as anchoring on the maximum possible payoff and attraction to the maximum possible payoff, do not suggest a change, given that the distribution of posted prices (particularly the maximum posted price) does not change. Finally, behavioral biases based on being a seller, such as “bad deal” aversion and the “buy-low sell-high” heuristic, do not suggest a change, given that the subject is still in the role of seller.

One behavioral bias that can explain the change in offers we see with our new protocol is framing effects. For instance, this protocol could induce a framing effect based on the repeated appearance of $1 in the instructions, which could make offering $1 a more salient action or even make $1 the reference point.

4.4.2 Decision-Making Noise and All-Or-Nothing Misperception

In addition to these behavioral biases, mistakes could arise in the BDM if agents have decision-making noise unrelated to their perception of the payoff function. Further, the movement in offers towards the dominant strategy with our new contingent protocol could be explained with a reduction in such decision-making noise.

However, using maximum likelihood estimation (MLE), we find that a representative agent model of decision-making noise based on logit errors fits the data significantly better (in a statistical sense) if it also allows for partial understanding of incentives, as in our model. We also find that partial payoff function perception fits the data significantly better than all-or-nothing payoff function recognition.\footnote{In the body of the text, we present the results for the baseline replication and contingent protocol, and in the appendix, we show that the same holds for the data from the CP experiment.}

As in CP, we first examine the possibility that choice mistakes are driven purely by decision-making noise by using maximum likelihood to estimate the “noise” parameter $\lambda$ that best explains offers. This parameter is taken from the Quantal Response Equilibrium (QRE) approach in which the likelihood of taking an action takes the form of a multinomial logit. A feature of this approach is that the frequency of taking an action is increasing its relative payoff.
To nest several models, we will use the function \( l_i(\gamma, \lambda) \), which is the likelihood of offer \( b_i \) if the agent has belief of the BDM \( \gamma \) and has noise parameter \( \lambda \). For the QRE specification,

\[
\ln l_i(\gamma, \lambda) = \ln \frac{e^{\lambda E[\text{payoff for } \gamma|b_i]}}{\sum_{k \in K} e^{\lambda E[\text{payoff for } \gamma|b_k]}}
\]  

(6)

where \( E[\text{payoff for } \gamma|b_i] = \frac{1}{p}(\theta b_i + .5\gamma p^2 + (1-\gamma)b_i\bar{p} - (1-.5\gamma)b_i^2) \) and \( K \) is the set of possible offers.

Thus, to estimate \( \lambda \) for a representative agent model with noise but no misperception, we set \( \gamma = 1 \) and find

\[
\arg \max \lambda \sum_{i \in I} \ln l_i(1, \lambda) = \arg \max \lambda \sum_{i \in I} \ln \frac{e^{\lambda E[\text{payoff for } \gamma=1|b_i]}}{\sum_{k \in K} e^{\lambda E[\text{payoff for } \gamma=1|b_k]}}
\]  

(7)

where \( b_i \) is the offer of subject \( i \) and \( I \) is set of subjects. We solve this problem using the Nelder-Mead method with 1,000 random started values, and standard errors were computed using 1,000 bootstrapping samples. As shown in Table 4, the parameter that best explains the data is 0.8040.

CP also estimate an all-or-nothing model of payoff function misrecognition in which there is a probability \( M \) that subjects believe they are facing the FPA. Specifically, they solve

\[
\arg \max \lambda, M \sum_{i \in I} \ln \left[(1 - M)l_i(1, \lambda) + ML_i(0, \lambda)\right]
\]  

(8)

We estimate their mixture model using the Nelder-Mead method with 1,000 random started values, and standard errors were computed using 1,000 bootstrapping samples. We estimate that 84.7% of subjects are playing as if they are facing a first price auction in the baseline replication and 36.9% in the contingent protocol. The estimate of \( \lambda \) is higher than with just noise: rising from 0.8040 to 2.1864, which means that less error is needed to explain the data (\( \lambda = 0 \) produces purely random choice).

To add partial payoff function perception, we make one small change to the CP estimation. Instead of estimating a mixture between no payoff function misrecognition \( (l_i(1, \lambda)) \) and full payoff function misrecognition \( (l_i(0, \lambda)) \), we estimate a mixture between no payoff function misperception \( (l_i(1, \lambda)) \) and partial payoff function misperception \( (l_i(\gamma, \lambda)) \). By allowing for a representative posterior that captures uncertainty, we add another parameter to the model. Thus, we solve

\[
\arg \max \lambda, M, \gamma \sum_{i \in I} \ln \left[(1 - M)l_i(1, \lambda) + ML_i(\gamma, \lambda)\right]
\]  

(9)

\(^{26}\)We follow CP by discretizing the space of offers, but because the increment of posted prices is $0.50, we use $0.50 instead of $0.10.
With this, we can estimate a very simple representative agent version of our model, which can be interpreted as a “representative belief” model. In principle, we could consider richer models, such as a representative information structure with two $\gamma$’s. In this case, $M$ could be interpreted as the probability of each belief.

We once again use the Nelder-Mead method with 1,000 random started values to perform our estimations, and compute standard errors using 1,000 bootstrapping samples. Using this mixture model, we find that 100% of subjects are classified as playing as if they are unsure of the mechanism payoffs in the baseline replication and the contingent protocol. The belief $\gamma$ that best explains the data is being 39.1% sure of the correct payoff function in the baseline replication and 66.2% in the contingent protocol. The estimate of $\lambda$ is once again higher than with just noise: rising from 0.8040 to 2.8786, which means that less error is needed to explain the data. The levels of error needed for the CP mixture model and the $\gamma$ mixture model are not statistically different.

Because these models are nested (the noise-only model in the CP all-or-nothing mixture model and the CP all-or-nothing mixture model in the $\gamma$ mixture model), we can use a likelihood ratio test to look for evidence of whether the fit of the model with more parameters is significantly better than the fit of the model with fewer parameters. With large samples, twice the difference in likelihoods should be distributed as a $\chi^2$ statistic with degrees of freedom equal to the difference in the number of parameters in the model. For one degree of freedom, the critical value for a significance level of 1% is 6.635.

For the baseline replication, twice the difference in likelihoods between the CP mixture model and noise-only model is 52.9294, which is well above 6.635, so the CP mixture model has significantly better fit. For the CP mixture model and the $\gamma$ mixture model, twice the difference is 8.9922, which is again above 6.635, so the $\gamma$ mixture model has significantly better fit. For the contingent protocol, the twice the difference in likelihoods between the CP mixture model and noise-only model is 19.695, so again the CP mixture model has significantly better fit. For the CP mixture model and the $\gamma$ mixture model, it is 26.088, so here too the $\gamma$ mixture model has significantly better fit.

5 Conclusion

In this paper, we take a standard model of imperfect perception – receiving a noisy mental signal of the environment – and use it to model an agent’s misperception of the BDM mechanism’s incentives. Our approach provides an as if representation for agents who may have trouble thinking through the mechanism’s complex payoff function.

We sharpen our model by assuming that mental signals are costly, and this generates
Table 4: Maximum likelihood estimates of different models. Standard errors are in parentheses.

(a) Baseline replication

<table>
<thead>
<tr>
<th></th>
<th>Noise-only model</th>
<th>CP mixture model</th>
<th>$\gamma$ mixture model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda$</td>
<td>0.8040</td>
<td>2.1864</td>
<td>2.8786</td>
</tr>
<tr>
<td></td>
<td>(0.4049)</td>
<td>(0.9865)</td>
<td>(0.5207)</td>
</tr>
<tr>
<td>90% conf. interval</td>
<td>[0.2197,1.5776]</td>
<td>[1.5052,4.4470]</td>
<td>[2.1203,3.8295]</td>
</tr>
<tr>
<td>$M$</td>
<td>0.8465</td>
<td>1.0000</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.1550)</td>
<td>(0.0000)</td>
<td></td>
</tr>
<tr>
<td>90% conf. interval</td>
<td>[0.5734,1.0000]</td>
<td>[1.0000,1.0000]</td>
<td></td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.3914</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0972)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>90% conf. interval</td>
<td></td>
<td>[0.2194,0.5352]</td>
<td></td>
</tr>
<tr>
<td>Avg. log likelihood</td>
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<td>-1.7929</td>
<td>-1.7693</td>
</tr>
<tr>
<td>Log likelihood</td>
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<td>-340.6568</td>
<td>-336.1607</td>
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</tbody>
</table>

(b) Contingent protocol

<table>
<thead>
<tr>
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<th>Noise-only model</th>
<th>CP mixture model</th>
<th>$\gamma$ mixture model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda$</td>
<td>2.3937</td>
<td>3.8515</td>
<td>4.1224</td>
</tr>
<tr>
<td></td>
<td>(0.6101)</td>
<td>(1.3467)</td>
<td>(0.7113)</td>
</tr>
<tr>
<td>90% conf. interval</td>
<td>[1.6090,3.6029]</td>
<td>[2.3677,6.7400]</td>
<td>[3.1602,5.4883]</td>
</tr>
<tr>
<td>$M$</td>
<td>0.3693</td>
<td>1.0000</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0961)</td>
<td>(0.0000)</td>
<td></td>
</tr>
<tr>
<td>90% conf. interval</td>
<td>[0.2521,0.5443]</td>
<td>[1.0000,1.0000]</td>
<td></td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.6615</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0494)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>90% conf. interval</td>
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<td>[0.5794,0.7445]</td>
<td></td>
</tr>
<tr>
<td>Avg. log likelihood</td>
<td>-1.8558</td>
<td>-1.8048</td>
<td>-1.7372</td>
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<tr>
<td>Log likelihood</td>
<td>-358.1783</td>
<td>-348.3308</td>
<td>-335.2868</td>
</tr>
</tbody>
</table>
comparative static predictions for the costs and benefits of more accurate perception. By reexamining the data from the CP experiment and generating data with new experiments, we are able to test these predictions, and we find that behavior is consistent with our theoretical predictions.

Because we leave the extensive form unchanged between our contingent protocol and baseline replication, the difference in mistakes between these experiments is plausibly related to a reduction in misperception of the payoff function. The remaining mistakes we observe with the contingent protocol could be due to lingering misperceptions or other behavioral biases.
References


Caplin, Andrew and Mark Dean (2013). “Behavioral implications of rational inattention with shannon entropy”. In: Working paper provided by NBER.


6 Appendix

6.1 Discrete Offers

In this section we derive the optimal offer strategy when offers are discrete, which is often the case in practice. First we examine the simple case of unitary increments in offers (in the units of $\theta$) and then we generalize to increments of size $z$.

6.1.1 Unitary Increments

We first derive the optimal strategy for the BDM case when offers are discrete. As in the continuous model, we assume that agent sells as long as the posted price $p$ is at least equal to the offered price $b$. Offering the own valuation $\theta$ is a weakly dominant strategy. $\theta + 1$ yields to the same payoff in expectation.

To see that $\theta$ is weakly dominant, consider a fixed offer price $b \leq \theta - 1$, and compare it to the payoff of offering $\theta$. The agent gets the same payoff as offering $\theta$ if the realization of $p$ lies in $[0, b - 1]$ (in which case gets $\theta$) or in $[\theta, \bar{p}]$ (in which case gets $p$). For the interval $[b, \theta - 1]$, the payoff of offering $\theta$ dominates the payoff of offering $b$. On the other hand, offering $b \geq \theta + 2$ there is a positive chance of making a negative payoff, so it is dominated by offering $\theta$. It is easy to see that $\theta + 1$ yields the same payoff in expectation, by writing the payoff and manipulating the expression.

On the other hand, in a FPA with discrete offers, the agent’s expected payoff from offering $b$ is

$$\sum_{p \in \{0,1,\ldots,\bar{p}\}} \theta \mathbb{1}[p < b] + b \mathbb{1}[p \geq b] = \theta \cdot \text{Prob}[p < b] + b \cdot \text{Prob}[p \geq b]$$

$$= \theta \cdot \text{Prob}[p \leq b - 1] + b \cdot (1 - \text{Prob}[p \leq b - 1])$$

Because $p$ is uniformly distributed on the finite set $\{0,1,\ldots,\bar{p}\}$ the former expression becomes

$$\theta \frac{b}{\bar{p} + 1} + b \left(1 - \frac{b}{\bar{p} + 1}\right)$$

Since $\bar{p} + 1 > 0$, we are interested in finding $b$ s.t. $b(\theta + \bar{p} + 1) - b^2$ is maximal. This can be rewritten as

$$-\left(b - \frac{\theta + \bar{p} + 1}{2}\right)^2 + \frac{(\theta + \bar{p} + 1)^2}{4}$$

which is a quadratic function of $b$ that reaches the maximum at $b^*$, with

$$b^* = \begin{cases} \frac{\theta + \bar{p} + 1}{2}, & \text{if } \theta + \bar{p} + 1 \text{ is even} \\ \left\{\frac{\theta + \bar{p}}{2}, \frac{\theta + \bar{p} + 1}{2}\right\}, & \text{if } \theta + \bar{p} + 1 \text{ is odd.} \end{cases}$$
With uncertainty about the payoff function, the expected payoff for offering \( b \) when agent assigns probability \( \gamma \) to the payoff function being the BDM is,

\[
\theta \text{Prob}[p < b] + E_p[(\gamma p + (1 - \gamma)b)\mathbb{1}[p \geq b]]
\]

which reduces to

\[
\theta \frac{b}{p + 1} + \gamma E_p[p\mathbb{1}[p \geq b]] + (1 - \gamma)b \left(1 - \frac{b}{p + 1}\right)
\]

Since \( E_p[p\mathbb{1}[p \geq b]] = \sum_{i=b}^{p} \frac{i}{p+1} = \frac{1}{2(p+1)} [p(p + 1) - (b - 1)b] \), we have

\[
\theta \frac{b}{p + 1} + \gamma \frac{1}{2(p+1)} (\tilde{p}^2 - b^2) + \gamma \frac{1}{2(p+1)} (b + \tilde{p}) - (1 - \gamma) \frac{b^2}{p + 1} + (1 - \gamma)b
\]

rearranging,

\[
- \left(1 - \frac{1 - \gamma}{p + 1} + \frac{2}{2(p+1)} \right)b^2 + \left(\frac{\theta}{p + 1} + \frac{\gamma}{2(p+1)} + 1 - \gamma\right)b + \frac{\gamma}{2(p+1)} (\tilde{p}^2 + \tilde{p})
\]

since this is a concave function in \( b \), we always have interior solution. By symmetry, it happens at either \( K = \frac{1}{2} \frac{\theta + \tilde{p} + (1 - \gamma)\tilde{p}}{2 - \gamma} = \frac{\theta + \tilde{p} - \gamma\tilde{p}}{2 - \gamma} + \frac{1}{2} \) if \( K \) is integer\(^{27}\); otherwise, at the closest integer. More explicitly,

\[
b^* = \begin{cases} 
[K], & \text{if } K - [K] \neq 0.5, \\
\{[K], [K] + 1\}, & \text{otherwise.}
\end{cases}
\]

Observe that if \( \gamma = 0, 1 \) we obtain the expected optimal offers for the BDM and FPA above derived.

6.1.2 Fractional Increments

We now consider the case where offers on a grid of mesh \( z \) on \([0, \tilde{p}]\) are allowed. In other words, agents can offer \( 0, z, 2z, 3z, ..., \tilde{p}. \)

This case is equivalent to solving the problem with unitary increments on \([0, \tilde{p}]\) and realized type \( \frac{\theta}{z} \) which yields the optimal offer \( b^* \) as in (10). In the original setting, the optimal offer corresponds then to the transformation \( b^*_z = zb^* \).

More explicitly, for the problem with mesh \( z \), define

\[
K' = \frac{\theta}{2} + \frac{\tilde{p}}{2} - \frac{\gamma\tilde{p}}{2 - \gamma} + \frac{1}{2}
\]

\(^{27}K \) is decreasing in \( \gamma \). Hence a tight upper bound for \( K \) is \( \tilde{p} + \frac{1}{2} \), reached when \( \theta = \tilde{p} \). In this case, it is optimal to offer either \( \tilde{p} \) or \( \tilde{p} + 1 \), the latter interpreted as no selling, which was a priori obvious.
Hence, the optimal offer price is given by

\[ b^*_z = \begin{cases} 
  z \lceil K' \rceil, & \text{if } K' - \lfloor K' \rfloor \neq 0.5, \\
  \{z \lfloor K' \rfloor, z \lceil K' \rceil\}, & \text{otherwise.}
\end{cases} \]

Observe also that as \( z \) goes to 0, the solution converges to the continuous case.

### 6.2 Alternative Payoff Functions

In this section, we show how the optimal offer strategy changes as we change the mechanism’s payoffs agents confuse with the BDM.

As these optimal offers show, the testable implications of our approach depend on the alternative payoff function that is selected. For instance, in some cases only offers at or below \( \theta \) are consistent, and in other cases only offers at or above \( \theta \) are consistent. Some cases even allow offers to be above the maximum posted price \( \bar{p} \).

<table>
<thead>
<tr>
<th>Alternative payment rule</th>
<th>Expected payoff</th>
<th>Map between offer and beliefs</th>
</tr>
</thead>
</table>
| Posted price \( p \) is paid, regardless of winning | \( E[\gamma(\theta 1[p < b] + p 1[p \geq b]) + (1 - \gamma)p] \) | \( b^*(\gamma) = \begin{cases} \theta, & \text{if } \gamma \in (0, 1], \\
  x \in \mathbb{R}, & \gamma = 0. \end{cases} \) |
| Offer \( b \) is paid, regardless of winning | \( E[\gamma(\theta 1[p < b] + p 1[p \geq b]) + (1 - \gamma)b] \) | \( b^*(\gamma) = \begin{cases} \theta + \frac{1 - \gamma}{\gamma} \bar{p}, & \text{if } \gamma \in (0, 1] \\
  \infty & \gamma = 0. \end{cases} \) |
| Paid theta only if offer is below posted price, nothing otherwise | \( E[\gamma(\theta 1[p < b] + p 1[p \geq b]) + (1 - \gamma)\theta 1[p \geq b]] \) | \( b^*(\gamma) = \begin{cases} \frac{\theta}{\gamma}, & \text{if } \gamma \in (0, 1] \\
  0, & \text{if } \gamma = 0. \end{cases} \) |
| Paid offer only if offer is below posted price, nothing otherwise | \( E[\gamma(\theta 1[p < b] + p 1[p \geq b]) + (1 - \gamma)b 1[p \geq b]] \) | \( b^*(\gamma) = \frac{\gamma \theta + (1 - \gamma)p}{2 - \gamma} \) |
| Paid posted price if offer below posted price, nothing otherwise | \( E[\gamma(\theta 1[p < b] + p 1[p \geq b]) + (1 - \gamma)p 1[p \geq b]] \) | \( b^*(\gamma) = \gamma \theta \) |

### 6.3 Additional Robustness Check: Within-Subject Comparisons

Because our robustness check was based on variation between subjects, we ran an additional robustness check in which 137 subjects first completed the BDM with \$0.01 increments.
and then either our baseline replication (with $0.50 increments) or our contingent protocol, separated by 5 months. This design allows us to perform a within-subject analysis of the impact of changes in the protocol. However, there is the possibility that learning occurred between rounds, so we do not use this as our primary robustness check and we do not include these subjects in any of our other analyses.

For the 74 subjects who completed the BDM with $0.01 increments and our baseline replication, there is an increase in offers of 0.081 on average, but this is not statistically different from 0 using a two-tailed t-test (p-value=0.3843). On the other hand, because our baseline replication has more offer prices at increments of $0.50, there is actually a 9.5% increase in the probability of offering near the value of the token, and this is statistically different from 0 (p-value=0.0073).

On the other hand, for the 63 subjects who completed the BDM with $0.01 increments and our contingent protocol, there is a decrease in offers on average (a drop of 0.075), but this is also not statistically different from zero using a two-tailed t-test (p-value=0.5588). At the same time, there is a much larger increase in the probability of offering near the value of the token with this protocol (23.8%), which is highly statistically different from a 0% increase (p-value<0.0001).

6.4 MLE Results for the CP Experiment

Here we apply the MLE strategy introduced in the body of the text to the two rounds of the CP experiment, and the results are similar to our results for the baseline replication and contingent protocol.

First, we estimate the noise-only model and mixture model from CP using the Nelder-Mead method with 1,000 random started values. Standard errors were computed using 1,000 bootstrapping samples. We find that 65.0% of subjects are classified as playing as if they are facing a first price auction on the first card and 41.1% on the second card.

Next, we estimate the representative agent version of our model, and we find that 84.4% of subjects are classified as playing as if they are unsure of the mechanism’s payoff function on the first card and 87.8% on the second card. While these rates are similar, the belief γ that best explains the data is being 41.1% sure of the correct payoff function on the first card and 61.9% on the second card.

Because these models are nested, we can use a likelihood ratio test to look for evidence of whether fits are significantly better. With large samples, twice the difference in likelihoods should be distributed as a χ² statistic with degrees of freedom equal to the difference in

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28For comparability, we follow CP by removing all offers over $\bar{p}$ and rounding offers to $0.10.$
the number of parameters in the model. For one degree of freedom, the critical value for a significance level of 1% is 6.635, so for both rounds, the CP mixture model has significantly better fit than the noise-only model, and the \( \gamma \) mixture model has significantly better fit than the CP mixture model.
Table 5: Maximum likelihood estimates of different models. Standard errors are in parentheses.

<table>
<thead>
<tr>
<th></th>
<th>Noise-only model</th>
<th>CP mixture model</th>
<th>γ mixture model</th>
</tr>
</thead>
<tbody>
<tr>
<td>λ</td>
<td>0.9833</td>
<td>4.4932</td>
<td>3.9626</td>
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<tr>
<td></td>
<td>(0.1609)</td>
<td>(0.9398)</td>
<td>(1.0408)</td>
</tr>
<tr>
<td>90% conf. interval</td>
<td>[0.7399,1.2649]</td>
<td>[3.2787,6.4129]</td>
<td>[3.1459,6.4437]</td>
</tr>
<tr>
<td>M</td>
<td>0.6499</td>
<td>0.8443</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0465)</td>
<td>(0.0798)</td>
<td></td>
</tr>
<tr>
<td>90% conf. interval</td>
<td></td>
<td>[0.5756,0.7302]</td>
<td>[0.7048,0.9704]</td>
</tr>
<tr>
<td>γ</td>
<td>0.4107</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0870)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>90% conf. interval</td>
<td></td>
<td></td>
<td>[0.2358,0.5244]</td>
</tr>
<tr>
<td>Avg. log likelihood</td>
<td>-4.0228</td>
<td>-3.8055</td>
<td>-3.7608</td>
</tr>
<tr>
<td>Log likelihood</td>
<td>-985.5835</td>
<td>-932.3379</td>
<td>-921.4038</td>
</tr>
</tbody>
</table>

(a) Round 1 (N=245)

<table>
<thead>
<tr>
<th></th>
<th>Noise-only model</th>
<th>CP mixture model</th>
<th>γ mixture model</th>
</tr>
</thead>
<tbody>
<tr>
<td>λ</td>
<td>1.2628</td>
<td>3.4536</td>
<td>2.6909</td>
</tr>
<tr>
<td></td>
<td>(0.2646)</td>
<td>(1.1599)</td>
<td>(0.5375)</td>
</tr>
<tr>
<td>90% conf. interval</td>
<td>[0.8928,1.7440]</td>
<td>[2.1001,6.0094]</td>
<td>[2.0811,3.6415]</td>
</tr>
<tr>
<td>M</td>
<td>0.4107</td>
<td>0.8781</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0561)</td>
<td>(0.1052)</td>
<td></td>
</tr>
<tr>
<td>90% conf. interval</td>
<td></td>
<td>[0.3253,0.5099]</td>
<td>[0.6968,1.0000]</td>
</tr>
<tr>
<td>γ</td>
<td>0.6188</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0711)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>90% conf. interval</td>
<td></td>
<td></td>
<td>[0.4947,0.7068]</td>
</tr>
<tr>
<td>Avg. log likelihood</td>
<td>-4.0013</td>
<td>-3.9198</td>
<td>-3.8824</td>
</tr>
<tr>
<td>Log likelihood</td>
<td>-985.3104</td>
<td>-948.5934</td>
<td>-939.5417</td>
</tr>
</tbody>
</table>

(b) Round 2 (N=242)
6.5 Experimental Instructions

Baseline Replication

You have one digital token that is worth $1 to you. You can sell it. Name your offer price: $____

A "posted" price will be drawn randomly between $0 and $3 (in increments of $0.50). Every possible posted price ($0, $0.50, $1, $1.50, $2, $2.50, and $3) has an equal chance of being selected.

If your offer price is at or below the posted price, then you sell your token at the posted price.

If your offer price is above the posted price, then you do not sell your token, but you do collect the $1 value of the token.

Contingent Protocol

You have one digital token that can be exchanged for $1 or sold to a computer bidder.

Here is how your bonus payment is determined:

1. You will name the minimum amount you are willing to sell your token (it might sell for more).

2. We will randomly select 1 of the 7 computer bidders listed below (each is equally likely to be selected).

3. You will keep your token if the minimum amount you are willing to sell it is above the computer’s bid. Otherwise, you will sell your token at the computer’s bid.

<table>
<thead>
<tr>
<th>Bidder</th>
<th>Bid</th>
<th>If your minimum amount is above their bid</th>
<th>Otherwise</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$0</td>
<td>$1</td>
<td>$0</td>
</tr>
<tr>
<td>B</td>
<td>$0.50</td>
<td>$1</td>
<td>$0.50</td>
</tr>
<tr>
<td>C</td>
<td>$1</td>
<td>$1</td>
<td>$1</td>
</tr>
<tr>
<td>D</td>
<td>$1.50</td>
<td>$1</td>
<td>$1.50</td>
</tr>
<tr>
<td>E</td>
<td>$2</td>
<td>$1</td>
<td>$2</td>
</tr>
<tr>
<td>F</td>
<td>$2.50</td>
<td>$1</td>
<td>$2.50</td>
</tr>
<tr>
<td>G</td>
<td>$3</td>
<td>$1</td>
<td>$3</td>
</tr>
</tbody>
</table>

Name the minimum amount you are willing to sell the token for: $____
Robustness Check

You have one digital token that is worth $1 to you. You can sell it. Name your offer price: $_____

A "posted" price will be drawn randomly between $0 and $3 (in increments of $0.01). Every possible posted price has an equal chance of being selected.

If your offer price is at or below the posted price, then you sell your token at the posted price.

If your offer price is above the posted price, then you do not sell your token, but you do collect the $1 value of the token.

Robustness Check (with Statement)

You have one digital token that is worth $1 to you. You can sell it.

The rule for selling the token is given below. It is a bit unusual, but its implications are straightforward. There is no way of gaming the rule, the BEST thing that you can do is to ask yourself how much you would be willing to exchange the token for, and then offer the number closest to that amount.

Name your offer price: $_____

A "posted" price will be drawn randomly between $0 and $3 (in increments of $0.01). Every possible posted price has an equal chance of being selected.

If your offer price is at or below the posted price, then you sell your token at the posted price.

If your offer price is above the posted price, then you do not sell your token, but you do collect the $1 value of the token.