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Strategic pricing with rational inattention to quality ☆

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ABSTRACT

Using a standard strategic pricing game, I determine how sellers set prices when facing buyers who are "rationally inattentive" to information about product quality. Two cases are studied: strategically sophisticated buyers who are rationally inattentive to exogenous information about quality and strategically naïve buyers who are rationally inattentive to strategic information about quality. In both cases, there exists an equilibrium where high quality sellers price high and low quality sellers mimic them by pricing high with a positive probability. This mimicking rate is uniquely identified and determines the informativeness of prices. In general, a drop in the marginal cost of attention results in *more* informative prices, but I identify conditions for which a drop in the marginal cost of attention can result in *less* informative prices.

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1. Introduction

In retail environments – both online and offline – where prices are simple and prominently displayed, the price of a product signals something about its quality. Consumers rely on the informational content of prices when other sources of information about quality, such as technical specifications and customer reviews, require cognitive effort to internalize. However, the informational content of prices is not given explicitly, and consumers may also find it cognitively demanding to understand the signaling power of prices.

In equilibrium, cognition plays two roles: not only is it important for assessing the informational content of prices, it is also important for ensuring the informational content of prices. If consumers are inattentive – either to exogenous sources of information about quality or to strategic sources of information about quality – firms with low quality products will be able to successfully charge the same price as firms with high quality products, which will result in uninformative prices.

I study the interplay between the informativeness of prices and consumer inattention by examining the equilibrium of a standard strategic pricing game that has been adapted to include inattention to quality. I use the "rational inattention" approach introduced by Sims (2003) to model inattention to quality in an abstract way. With this approach, agents can adjust their attention on three margins: when they pay attention (extensive margin), how much they pay attention (intensive margin), and to which information they pay attention. Agents limit their attention because information generates costs that are measured in terms of Shannon entropy.

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To isolate the impact of rational inattention to quality on pricing behavior, I focus on a simple market setting in which there is one seller, one buyer, and one product of uncertain quality. This game can be interpreted as a representative sales encounter in a market with a monopolist and many buyers who interact with the monopolist independently. In the first move of this game, nature determines the quality of the seller's product according to a commonly known probability distribution. Next, the seller, who knows the quality of the product, offers a take-it-or-leave-it price to the buyer, who does not know the quality of the product.[1] Finally, the buyer, who knows the offered price, attends to information about product quality (either exogenous or strategic), and then decides whether or not to accept the seller's offer. The seller wants to sell the product at the highest possible price, and the buyer only wants to accept the offer if the price is sufficiently low given quality.

For both forms of inattention, there exists a semi-separating (or partial pooling) equilibrium where sellers of high quality products charge a high price and sellers of low quality products sometimes mimic them by charging a high price. I characterize this mimicking equilibrium by identifying the unique rate at which low quality sellers mimic high quality sellers as a function of model parameters, including the marginal cost of attention.

This rate of mimicking dictates the informational content of prices, and we might expect that a decrease in the marginal cost of attention would always lead to less mimicking, which increases the informativeness of prices. However, I demonstrate that for strategically sophisticated buyers who are inattentive to exogenous sources of information about quality, a drop in the marginal cost of attention can actually result in *more* mimicking, which *reduces* the informativeness of prices.

A necessary condition for this to happen is that not purchasing (the buyer's outside option) must be relatively more attractive than blindly purchasing from a random seller. In this case, if attentional costs are so high that buyers do not bother to distinguish between low and high quality sellers, then low quality sellers cannot mimic too often. If they do, then buyers will just take the outside option.[2] As the marginal cost of attention falls, two forces work in opposite directions: 1) buyers get better informed about quality before they purchase, which increases the probability that low quality sellers do not make a sale when they mimic, and 2) buyers are less likely to take the outside option, which increases the probability that low quality sellers make a sale when they mimic. When the price that high quality sellers charge is high enough, the latter force dominates, so low quality sellers end up mimicking more often as the attentional cost parameter falls.

To get an intuition why the informativeness of prices might decrease when the costs of attention fall, consider the following example, which is based on buying a used printer cartridge online. In this example, a natural outside option is to instead buy a new printer cartridge of known quality. Assume that it is hard to distinguish high and low quality used cartridges, so that if a consumer sees a higher price for a used printer cartridge, then it is not worth putting in attentional effort (for instance, in reading reviews of the company or product) and the consumer leans towards buying a new printer cartridge instead of a used one. If it gets less costly to learn about the quality of used cartridges, buying a used cartridge becomes relatively more attractive. This will allow a firm selling a used printer cartridge of low quality to set a high price more often, which decreases the informativeness of prices.

Of course, care should be taken when making generalizations from highly stylized models, but this comparative static could provide a potential explanation for a puzzle from the marketing literature. Garbarino and Oromulu (2012) document that the correlation between price and quality has not increased for some product categories with the advent of the Internet, even though it is believed that the Internet has made the costs of information search lower.

This paper contributes to two different literatures, which are discussed in more detail in the next section. First, this paper contributes to an expanding literature around rational inattention theory by presenting one of the first applications to games of the most flexible version of rational inattention. Second, this paper contributes to a large literature on strategic naïveté. To the best of my knowledge, this paper presents the first use of rational inattention to model how an agent's level of strategic naïveté responds to incentives.

In section 2, I provide a review of the related literature. In section 3, I describe the model. For the case of strategically sophisticated buyers who are inattentive to exogenous information about quality, I characterize the mimicking equilibrium in closed form in section 4 and explore the impact of attentional costs on the informativeness of prices in this equilibrium in section 5. For the case of strategically naïve buyers who are inattentive to strategic information about quality, I characterize a mimicking equilibrium in closed form in section 6. Section 7 concludes.

2. Related literature

This section describes two literatures to which this paper contributes: rational inattention theory and strategic naïveté.

2.1. Rational inattention theory

Sims (2003) introduces rational inattention theory in order to model the constraints that agents face in processing readily available information. It is based on classic works in the information theory literature which describe a physical constraint

[1] My assumption that the buyer is ex ante uninformed about product quality is appropriate for durable goods that are rarely purchased and change features over time, such as computers, cell phones, and air conditioners.

[2] This does not occur when the costs of attention are low or when the outside option is less attractive than blindly purchasing from a random seller.

on the flow of information. This constraint, called the Shannon capacity or Shannon channel, determines the amount of uncertainty (entropy) that can be reduced by a message, and it is often interpreted as a cognitive limitation (Wiederholt, 2010). Some papers assume that the Shannon capacity is fixed, whereas other papers (such as this one) assume that the Shannon capacity can be increased at a cost.[3] Subject to this constraint, agents can choose any distribution from which they draw an informative signal. For tractability, many models have assumed a Gaussian relationship in the signal structure and a linear-quadratic utility function, but recent work (including this paper) allows for more flexibility by using general signal structures and utility functions (see Sims, 2006 for a discussion). Recently, Caplin and Dean (2013), Matějka and McKay (2014), Steiner et al. (2015) have shown that in individual choice models with a finite number of states, this approach can yield tractable characterizations of behavior. de Oliveira (2014) finds axioms on preferences for sets of acts that characterize such models.

This paper joins Gentzkow and Kamenica (2014), Matějka (2015), Yang (2015a), and Yang (2015b), as one of the first applications to games of the most flexible version of rational inattention (with no restrictions on signal structures). As in this paper, Gentzkow and Kamenica (2014), Yang (2015a), and Yang (2015b) assume that Shannon capacity can be increased at a cost.[4] Like Yang (2015a), I use a binary action setup with sequential moves and include only one rationally inattentive agent (the second mover). However, a substantial difference in my paper is that the first mover is informed, so that their action choice (here prices) can reveal information. Hence, my model opens the door to studying the interaction between the informativeness of prices and rational inattention.

Several subsequent papers have added to this growing literature. Ravid (2014) considers a bargaining game in which an informed proposer makes repeated take-it-or-leave-it offers to a rationally inattentive responder. Boyaci and Akçay (2016) also consider a similar model to this paper, but instead examine the case where buyers are unable to infer quality from price, no matter how much attention they pay. Gomes and Martin (2016) investigate equilibrium behavior when sellers do not know the valuations of buyers before they set the price for a product.

2.2. Strategic naïveté

There is a long literature in economics on limited strategic thinking (see Crawford et al., 2013 for a recent review). One branch of this literature is on level-k thinking, which was introduced by Nagel (1995) and Stahl and Wilson (1995). A related branch is the cognitive hierarchy approach of Camerer et al. (2004), which allows for best responding to a distribution of level-k types. Another branch is the cursed equilibrium approach proposed by Eyster and Rabin (2005).

A small number of papers have considered how strategic naïveté might arise endogenously. For instance, Esponda (2008) considers a learning process that leads to incorrect beliefs. Choi (2012) show that cognitive hierarchy can be the outcome of optimal choices in a network.

Most closely related to this paper are the papers by Alaoui and Penta (2015), who model agents that weight the costs and benefits of additional levels of reasoning, and by Gabaix (2012), who allows strategic agents to form incorrect beliefs because of the cost of forming correct beliefs. In the later paper, costs are modeled using a Sparsity-based approach. To the best of my knowledge, this the first paper to use entropic costs to endogenize strategic naïveté.

3. A model of strategic pricing

To isolate the impact of rational inattention, I add entropic information costs to a simple and standard sequential pricing game. This modified game is described in this section, and the corresponding equilibrium is defined in later sections.

3.1. The sales encounter

In the standard sequential pricing game, one seller and one buyer are engaged in a one-off sales encounter, which is represented by the game tree presented in Fig. 1. Nature moves first by determining the product's quality level $\theta \in \Theta = \{\theta_L, \theta_H\}$, where $\theta_L, \theta_H \in \mathbb{R}_{++}$ and $\theta_L < \theta_H$. The seller knows the realized quality level, but the buyer does not.[5] However, the ex-ante probability of each quality level is commonly known and is given by $\delta(\theta) \in (0, 1)$.

Next, the seller chooses a price $p \in P = \{p_L, p_H\}$, where $p_L, p_H \in \mathbb{R}_{++}$ and $p_L < p_H$, which is a take-it-or-leave-it offer for the product.

A market is defined by the triple (δ, Θ, P). Many forces could impact market parameters. For example, new technologies could alter the probability a product is of high quality or improve the absolute quality levels.

[3] Examples of the latter approach include Caplin and Dean (2013), de Oliveira (2014), Gentzkow and Kamenica (2014), Matějka and McKay (2014), Ravid (2014), Steiner et al. (2015). According to de Oliveira (2014), these two approaches are "locally" equivalent in the sense that "when solving for the decision maker optimal choice of information using Sims' formulation, the Lagrangian multiplier effectively turns the constraint into a linear representation".

[4] In addition, Gentzkow and Kamenica (2014) show that the concavification approach of Kamenica and Gentzkow (2011) can be applied if signals have Shannon costs, which is why attentional strategies can be written in terms of the posterior beliefs they generate.

[5] This is a suitable assumption when thinking about infrequently purchased products, such as durable goods that have features that change over time.

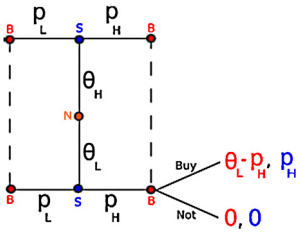

Fig. 1. The standard game tree.

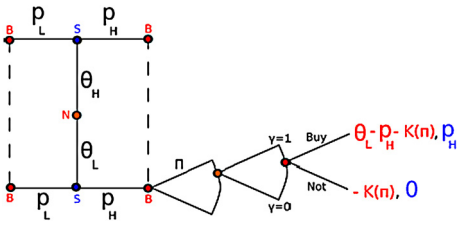

Fig. 2. The modified game tree.

3.2. Payoffs to the buyer and seller

The buyer has the following "purchase" utility function:

$$U(\theta, p, x) = x(\theta - p)$$

where $x = 1$ if the buyer chooses to purchase the product and $x = 0$ if not. This quasilinear utility function reflects a buyer who is balancing the quality and price of a good, which is suitable when prices do not have a big effect on wealth (see Vives, 2001). The utility gained from not purchasing the product can be interpreted as the utility gained from selecting an outside option, which has been normalized to zero. The buyer is a risk neutral expected utility maximizer.

To focus the analysis on nontrivial parameter values, I assume that

$$\theta_H > p_H \text{ and } p_L = \theta_L$$

With these assumption, the buyer wants to accept high quality offers and reject low quality offers at p_H, and all offers are accepted at price p_L.[6]

The seller has the following profit function:

$$V(\theta, p, x) = xp$$

The seller is also a risk neutral expected profit maximizer. This profit function reflects a seller who does not have reputation concerns or marginal costs of production. However, these features can be readily added to the model.

3.3. Adding rational inattention to quality

In the standard game, the buyer makes a purchase decision based entirely on her strategic belief μ, which is based on the information about quality that is signaled by price. However, as I argued previously, buyers may have access to other sources of information about quality or have trouble inferring what price implies about quality. To allow for these possibilities, I modify the standard sequential pricing game to include rational inattention to quality. So that price can continue to signal something about quality, I assume that the buyer attends to information (either exogenous or strategic) about quality *knowing* the seller's offered price.[7] With this addition, the standard game tree represented in Fig. 1 expands into the modified game tree represented in Fig. 2.

Following rational inattention theory, the buyer is uncertain about some state ω in a finite state space Ω, where $\nu(\omega)$ is the prior probability of state ω. The buyer chooses an information structure π, which stochastically generates a posterior belief $\gamma \in \Gamma$, where $\gamma(\omega)$ is the probability of state ω in Ω. In the case of inattention to exogenous information, the state is the quality level θ, and in the case of inattention to strategic information, the state is the strategic belief μ. In both cases, the buyer uses the posterior belief γ in deciding whether to buy the product or not.

[6] If sellers are allowed to choose any price $p \in P = (0, \bar{p}] \subset \mathbb{R}_{++}$ where $\theta_H > \bar{p} > \theta_L$ and buyers are strategically sophisticated, then there always exists an equilibrium of the modified game where sellers choose just two prices: $p_H = \bar{p}$ and $p_L = \theta_L$.

[7] The assumption that buyers cannot commit to their attention strategy before learning the product's price is realistic for many market settings and allows prices to have an incentive effect on attentional effort.

Technically, π is a function that maps the state into $\Delta(\Gamma)$, the set of probability distributions over Γ that have finite support, so that

$$\pi : \Omega \to \Delta(\Gamma)$$

Let Π denote the set of all such functions, $\pi(\gamma)$ be the unconditional probability of posterior $\gamma \in \Gamma$, $\pi(\gamma|\omega)$ be the probability of posterior γ given state ω, and $\Gamma(\pi) \subset \Gamma$ denote the support of a given π. I limit the set of information structures to those in $\Pi(\nu) \subset \Pi$ that generate correct posteriors for a given prior belief ν, so that

$$\Pi(\nu) = \left\{ \pi \in \Pi | \forall \gamma \in \Gamma(\pi), \forall \omega \in \Omega, \gamma(\omega) = \frac{\nu(\omega)\pi(\gamma|\omega)}{\sum_{\omega \in \Omega} \nu(\omega)\pi(\gamma|\omega)} \right\}$$

As shown in Kamenica and Gentzkow (2011) for costless information structures and in Gentzkow and Kamenica (2014) for costly information structures of the form considered in this paper, working directly with posteriors is without loss of generality, as there always exists a signal process that will generate a given set of posteriors. Caplin and Martin (2011) and Caplin and Dean (2013) also follow the approach of working directly with posterior beliefs.

Further, I assume that attention is costly, that attentional costs are separable in the utility function, and that these costs are determined using the Shannon entropy of the corresponding information structure. According to this approach, a posterior is more costly if it reduces more uncertainty. Formally, each information structure $\pi \in \Pi(\nu)$ has a cost in expected utility units that is assigned by the function

$$K(\pi, \lambda, \nu) = \lambda \left(\left[\sum_{\gamma \in \Gamma(\pi)} \pi(\gamma) \sum_{\omega \in \Omega} [\gamma(\omega) \ln(\gamma(\omega))] \right] - \sum_{\omega \in \Omega} [\nu(\omega) \ln(\nu(\omega))] \right)$$

where $\lambda \in \mathbb{R}_{++}$ is a linear cost parameter, which is interpreted as the marginal cost of attention. This functional form produces a u-shaped cost for each posterior, which increases symmetrically towards being certain of the state ω. In addition, the information structure that returns the prior ν as the posterior belief has a cost of zero. In other words, if there is no attention (and thus no information), then there is no attentional cost.

Adding rational inattention to quality to a market (δ, Θ, P) produces a game $G := (\delta, \Theta, P, \lambda)$. Everything about the structure of the game is common knowledge to both the seller and the buyer. An implicit assumption is that the seller knows the marginal cost of attention for the buyer. Rational inattention theory does not allow for a representative agent, so if there is heterogeneity in attentional cost parameters, a seller would need to know the attentional cost parameter of each buyer and be able to adjust their pricing strategy accordingly. In practice, firms may use data on purchasing patterns and demographic characteristics in an attempt to estimate the attentional costs of each consumer.

4. Strategically sophisticated buyer: equilibrium

In this section, I consider a strategically sophisticated buyer who has access to different exogenous sources of information about the quality of the product: physical inspection, information provided on the packaging, customer reviews, advertisements, etc. However, these information sources may require attentional effort to internalize, so the buyer may be inattentive to them.

Sims (2003) uses rational inattention theory to model inattention to high frequency information about macroeconomic variables, but this approach is also suitable for such exogenous information sources about product quality, as individuals make many consumption decisions per day, and there is often more information about product quality than consumers have time to process.[8] This is especially true for online commerce, where the sources of information about product quality have expanded exponentially and the corresponding information has become increasingly rich and dynamic.

In this case, the states Ω that the buyer is rationally inattentive about are the quality levels Θ. Because the buyer is strategically sophisticated, the prior probability ν over states Ω is summarized by the strategic belief μ over qualities Θ, which is formed correctly knowing price p.

Also, because the buyer is strategically sophisticated, the equilibrium concept I employ is mixed strategy perfect Bayesian equilibrium (PBE). The seller has pricing strategy $\sigma(p|\theta)$, which is the probability of setting price p for quality level θ. The buyer has an attention strategy π_p, which is the information structure selected for price p,[9] and purchasing strategy $\alpha(\gamma, p)$, which is the probability of purchasing for posterior γ and price p. Finally, the buyer has beliefs $\mu(\theta|p)$ of the probability of quality level θ when the price is p.

For a game G, a mixed strategy PBE is a 4-tuple $(\widehat{\sigma}, \widehat{\pi}, \widehat{\alpha}, \widehat{\mu})$ that satisfies seller optimality, buyer optimality, and Bayesian beliefs:

[8] In addition, laboratory experiments have found subject behavior that is consistent with rational inattention theory in settings with static information sources (Martin, 2015).

[9] Note that the agent does not mix over attention strategies. This is without loss of generality as Caplin and Dean (2013) show it is not optimal to mix over information structures with a Shannon cost function, which follows from the strict concavity of the log function.

- Seller optimality
  - $\forall\theta \in \Theta$ and $\forall p \in P$, $\widehat{\sigma}(p|\theta) > 0$ implies

$$p \in \arg\max_{p\in P} \sum_{\gamma\in\Gamma(\widehat{\pi}_p)} \widehat{\pi}_p(\gamma|\theta)\widehat{\alpha}(\gamma, p)p$$

- Buyer optimality
  - $\forall p \in P$ and $\forall\gamma \in \Gamma(\widehat{\pi}_p)$, $\widehat{\alpha}(\gamma, p) > 0$ implies

$$1 \in \arg\max_{x\in\{0,1\}} x\sum_{\theta\in\Theta}[\gamma(\theta)(\theta - p)]$$

  and $\widehat{\alpha}(\gamma, p) < 1$ implies

$$0 \in \arg\max_{x\in\{0,1\}} x\sum_{\theta\in\Theta}[\gamma(\theta)(\theta - p)]$$

  - $\forall p \in P$,

$$\widehat{\pi}_p \in \arg\max_{\pi\in\Pi(\widehat{\mu}(\cdot|p))} \sum_{\gamma\in\Gamma(\pi)} [\pi(\gamma)\widehat{\alpha}(\gamma, p)\sum_{\theta\in\Theta}[\gamma(\theta)(\theta - p)]] - K(\pi, \lambda, \widehat{\mu}(\cdot, p))$$

- Bayesian beliefs on path
  - If $\widehat{\sigma}(p|\theta) > 0$ for some $p \in P$, then

$$\widehat{\mu}(\theta|p) = \frac{\delta(\theta)\widehat{\sigma}(p|\theta)}{\sum_{\hat{\theta}\in\Theta}[\delta(\hat{\theta})\widehat{\sigma}(p|\hat{\theta})]}$$

4.1. Mimicking equilibrium

There always exists a "mimicking" equilibrium in which the high quality seller always charges a high price of p_H, and the low quality seller mimics the high quality seller by charging this price with a certain probability and otherwise charges a low price p_L. In this mimicking equilibrium, the attention strategy of the buyer makes a low quality seller indifferent between pricing high or low, and the rate at which a low quality seller prices high makes this attention strategy optimal. As a result, the prices that sellers offer are perfectly balanced with the attentional effort of buyers. The following theorem establishes that there always exists such an equilibrium and that for a given game G, the rate of mimicking is uniquely pinned down.

Theorem 1. *For any game G, there exists a mixed strategy PBE in which high quality sellers set price p_H with probability 1 and low quality sellers set price p_H with a unique probability η and otherwise set price p_L. Formally, this equilibrium is defined as:*

- *Seller pricing strategy*

$$\widehat{\sigma}(p_H|\theta_H) = 1, \widehat{\sigma}(p_H|\theta_L) = \eta, \widehat{\sigma}(p_L|\theta_L) = 1 - \eta$$

- *Buyer attention strategy*

$$\widehat{\pi}_{p_H}(\gamma^1) = \min\left\{\frac{\widehat{\mu}(\theta_H|p_H) - \gamma^0(\theta_H)}{\gamma^1(\theta_H) - \gamma^0(\theta_H)}, 1\right\}, \widehat{\pi}_{p_H}(\gamma^0) = 1 - \widehat{\pi}_{p_H}(\gamma^1), \widehat{\pi}_p(\gamma^*) = 1 \text{ if } p \neq p_H$$

- *Buyer purchasing strategy*

$$\widehat{\alpha}(\gamma^1, p_H) = 1, \widehat{\alpha}(\gamma^0, p_H) = 0, \widehat{\alpha}(\gamma^*, p_L) = 1$$

- *Buyer beliefs*

$$\widehat{\mu}(\theta_H|p_H) = \frac{\delta(\theta_H)}{\delta(\theta_H) + (1 - \delta(\theta_H))\eta}, \widehat{\mu}(\theta_H|p) = 0 \text{ if } p \neq p_H$$

- *Optimal posteriors*

$$\gamma^*(\theta_H) = 0, \gamma^0(\theta_H) = \min\left\{\frac{1 - \exp\left(\frac{\theta_L - p_H}{\lambda}\right)}{\exp\left(\frac{\theta_H - p_H}{\lambda}\right) - \exp\left(\frac{\theta_L - p_H}{\lambda}\right)}, \widehat{\mu}(\theta_H|p_H)\right\},$$

$$\gamma^1(\theta_H) = \max\left\{\exp\left(\frac{\theta_H - p_H}{\lambda}\right)\gamma^0(\theta_H), \widehat{\mu}(\theta_H|p_H)\right\}$$

- *Mimicking rate*

$$\eta = \min\left\{\frac{\delta(\theta_H)}{1-\delta(\theta_H)}\frac{(1-\gamma^0(\theta_H))(1-\gamma^1(\theta_H))}{\gamma^0(\theta_H)(1-\gamma^1(\theta_H))+\frac{p_L}{p_H}(\gamma^1(\theta_H)-\gamma^0(\theta_H))}, 1\right\}$$

Proof of this theorem is in Appendix A.

The attention strategy for buyers is quite simple – it produces two posterior beliefs when two actions are taken and one posterior belief when one action is taken. At a high price, buyers always purchase the good at one posterior (γ^1) and at the other posterior, buyers never do (γ^0). At a low price, buyers always purchase the good, so there is just one posterior (γ^*).

These posterior beliefs are given explicitly and are only a function of the model parameter values – not the seller's pricing strategy. The seller's pricing strategy only impacts how likely the buyer is to reach each posterior.

4.2. Rational inattention in equilibrium

Matějka and McKay (2014) show how to solve for the choice probabilities of a rationally inattentive decision maker. These choice probabilities take the form of generalized Logit demand probabilities. Caplin and Dean (2013) instead show how to solve for the agent's posterior beliefs and demonstrate that these posterior beliefs satisfy a particular log-likelihood ratio called the Invariant Likelihood Ratio (ILR).[10]

Both approaches can be used to solve the equilibria of games with rationally inattentive agents. For instance, Matějka and McKay (2012) use the choice probabilities approach. Instead, in this paper I use posterior beliefs to characterize the mimicking equilibrium.

The posterior beliefs approach demonstrates several useful properties of rational inattention in producing and establishing this equilibrium: convexity, log-linearity, and prior separability.

First, convexity of the cost function with respect to posterior beliefs helps to ensure that the attention strategy for buyers produces just two posterior beliefs when two actions are taken and one posterior belief when one action is taken. Second, the log-linear form of the cost function generates a closed-form solution for the optimal posteriors because the first-order conditions with respect to the posteriors are relatively simple. Third, because the costs of the strategic beliefs are separable in the cost function, optimal posteriors are locally invariant to changes in strategic beliefs. Thus, as the seller strategy varies, the optimal posteriors stay the same. As a result, the conditional demand for low quality products changes monotonically with strategic beliefs, so there is just one mimicking rate at which low quality sellers are indifferent between pricing high and low.

5. Strategically sophisticated buyer: attentional costs

The main comparative static of interest is how attentional costs impact the informativeness of prices, which is driven in this equilibrium by the mimicking rate η. For some games, when the marginal cost of attention is high enough, an increase in the marginal cost of attention actually decreases the probability that low quality sellers mimic. The following proposition expresses this formally.

Proposition 1. *For the equilibrium given in Theorem 1,*

- *If* $\delta(\theta_H) > \frac{p_H-\theta_L}{\theta_H-\theta_L}$, *then there exists* λ^* *such that for every* $\lambda \geq \lambda^*$, *there is complete mimicking:* $\eta = 1$.
- *If* $\delta(\theta_H) < \frac{p_H-\theta_L}{\theta_H-\theta_L}$, *then there exists* λ^{**} *such that for every* $\lambda \geq \lambda^{**}$, *there is less than complete mimicking:* $\eta > 0$ *and* $\eta < 1$.
  - *Further, if* $p_H > 2p_L$, *then for some* λ^{***}, *mimicking is strictly decreasing in* λ *for all* $\lambda > \lambda^{***}$.

Proof of this proposition is in Appendix A and immediately follows from two facts (both established in the proof). First, the uncapped rate of mimicking converges to $\frac{\delta(\theta_H)}{1-\delta(\theta_H)}\frac{\theta_H-p_H}{p_H-\theta_L}$ in the limit as the marginal cost of attention λ goes to infinity. Thus, if the uncapped rate of mimicking converges above 100% (if $\delta(\theta_H) > \frac{p_H-\theta_L}{\theta_H-\theta_L}$), then the capped rate of mimicking will converge to 100% also, and if the uncapped rate of mimicking converges below 100% (if $\delta(\theta_H) < \frac{p_H-\theta_L}{\theta_H-\theta_L}$), then the capped rate will also converge below 100%. Second, the uncapped rate of mimicking is decreasing in the limit as λ goes to infinity if (and only if) $p_H > 2p_L$.

[10] Gentzkow and Kamenica (2014) show how the concavification of values with respect to prior beliefs can be used to determine when persuasion is beneficial when information structures carry entropic costs, and Caplin and Dean (2013) show how the concavification of values with respect to prior beliefs for each posterior belief can be used to determine optimal attention with entropic costs.

5.1. Comparison to fixed information structures

To get an intuition for this proposition, it is helpful to compare the mimicking equilibrium of the modified game with the equilibrium that would result from two fixed information structures: a fully informative information structure and a fully uninformative information structure.

First, consider the case of a fully informative information structure, which can be interpreted as full attention (the standard case). If buyers are fully informed about product quality, then sellers with high quality products always price high and low quality products never price high. With rational inattention, as the marginal cost of attention λ goes to 0, the probability of mimicking η converges to 0 in any mimicking equilibrium. In other words, the mimicking equilibrium with rational inattention converges to the equilibrium of the full attention version of this game, in which there is full separation and prices are perfectly informative.

Next, consider the case of a fully uninformative information structure, which can be interpreted as no attention. Clearly, when there is no information about quality beyond price, low quality sellers will always want to mimic by pricing high. However, whether a mimicking equilibrium exists where high quality sellers charge the highest price and low quality sellers always mimic them when there is no attention depends on the parameters of the game.[11]

If purchasing when all sellers are pooled at the high price is better than taking the outside option, then for any fully uninformative information structure, an equilibrium exists where all sellers pool at the high price. This occurs when $\delta(\theta_H) > \frac{p_H - \theta_L}{\theta_H - \theta_L}$, which mirrors the conditions for which the rational inattention mimicking equilibrium converges to complete mimicking ($\eta = 1$). In other words, the rational inattention mimicking equilibrium converges to the equilibrium of the no attention version of this game for these parameter values.

However, for fixed no attention and bad fundamentals (so that the buyer prefers to take the outside option when there is full pooling at the high price), there will be no equilibrium at the high price because there is no way for buyers to separate low and high quality sellers. However, no matter how high the marginal cost of attention gets, there always exists a mimicking equilibrium with rational inattention, even when the fundamentals are bad. The buyer will pay just enough attention and the low quality seller will mimic just enough to keep things in balance.[12]

Further, if the difference between high and low prices is big enough, as the marginal cost of attention goes to infinity, the low quality seller will mimic less and less. In this case, even though a higher marginal cost of attention leads buyers to be less informed about quality, low quality sellers will mimic less because the probability of purchasing will go down.

6. Strategically naïve buyer: equilibrium

In this section, I consider a strategically naïve buyer who has cognitive difficulty in determining what prices imply about product quality. As a consequence, the buyer is uncertain about μ, his beliefs of high quality after observing price p. If the buyer was strategically sophisticated, he would be sure that the probability of high quality was $\widehat{\mu}$, the correct one given the seller's strategy and Bayes' rule.

In the language of the model presented in section 3, this means the state ω that the buyer is inattentive about is his strategic belief μ. The buyer's uncertainty about μ is given by the distribution ν over states. For a strategically sophisticated buyer, $\nu(\widehat{\mu})$ would be equal to 1. But because the buyer is strategically naïve, the distribution over strategic beliefs is not formed correctly.

Instead, I consider a buyer who has the following incorrect beliefs over μ. The buyer believes that with χ probability, μ will be equal to δ, the unconditional probability of high quality. As a result, the probability of δ is given by $\nu(\delta) = \chi$. The buyer believes that with the remaining probability, μ will be equal to the correct strategic beliefs, given by $\widehat{\mu}$. Thus, $\nu(\widehat{\mu}) = 1 - \chi$. This formulation of buyer beliefs is consistent with beliefs in a cursed equilibrium of this game (Eyster and Rabin, 2005).

However, as in the model presented in section 3, buyers can reduce their uncertainty by gathering signals about the state μ. Because the realized state is always $\widehat{\mu}$, with a perfectly informative signal structure, the buyer will have posterior beliefs that are correct. Thus, this model embeds χ-cursedness and full rationality as extreme cases.

Given the signal he receives, the buyer forms posterior beliefs γ, which he bases his purchase decision on. This posterior puts some probability on the unconditional probability of high quality δ, given by $\gamma(\delta)$, and some probability on the correct strategic beliefs about high quality $\widehat{\mu}$, given by $\gamma(\widehat{\mu})$. Loosely speaking, γ represents the new level of cursedness of the buyer after he attends to the strategic content of prices.

The buyer does not realize that his prior beliefs ν are incorrect, so he does not realize that given anything less than a fully informative signal structure, his posterior beliefs will also be incorrect. As a consequence, the buyer chooses an optimal information structure based on incorrect estimates of the costs of attention.

[11] Because the conditional demands are the same for high and low quality sellers when there is no attention, mimicking equilibria can exist where high quality sellers also mix between high and low prices. However, once there is even a small amount of information about quality available, this possibility disappears.

[12] The fact that the mimicking equilibrium always exists, even for very high marginal costs of attention, is not just because information is endogenous in the modified game. If entropic cost are replaced with being fully informed at a fixed cost (as in Bester and Ritzberger, 2001), then the most informative equilibrium characterized in this paper does not exist at higher information costs.

Given information structure π, the buyer believes that he will arrive at posterior γ with probability $\pi(\gamma) = \chi\pi(\gamma|\delta) + (1-\chi)\pi(\gamma|\widehat{\mu})$, which is the unconditional probability of receiving a signal that produces posterior γ if his prior χ is correct. However, because the true state is always $\widehat{\mu}$, the true likelihood of receiving a signal that produces posterior γ is $\pi(\gamma|\widehat{\mu})$.

For the seller, all that matters is how likely the buyer is to receive a signal that leads him to purchase the product. Technically, this comes from multiplying the probability the buyer reaches a posterior γ for information structure π, which is given by $\pi(\gamma|\widehat{\mu})$, and the probability he purchases the product at that posterior, which is given by $\alpha(\gamma, p)$. This is reflected in the seller's optimality condition below.

The equilibrium concept I employ is an extension of (perfectly) cursed equilibrium. The seller has pricing strategy $\sigma(p|\theta)$, which is the probability of setting price p for quality level θ. The buyer has an attention strategy π_p, which is the information structure selected for price p, and purchasing strategy $\alpha(\gamma, p)$, which is the probability of purchasing for posterior γ and price p.

For a game G and cursedness χ, a Rationally Inattentive Cursed Equilibrium (RICE) is a 3-tuple $(\widehat{\sigma}, \widehat{\pi}, \widehat{\alpha})$ that satisfies seller optimality, buyer optimality, and Bayesian beliefs (when not cursed):

- Seller optimality
  - $\forall\theta \in \Theta$ and $\forall p \in P$, $\widehat{\sigma}(p|\theta) > 0$ implies

$$p \in \arg\max_{p\in P} \sum_{\gamma\in\Gamma(\widehat{\pi}_p)} \widehat{\pi}_p(\gamma|\widehat{\mu})\widehat{\alpha}(\gamma, p)p$$

- Buyer optimality
  - $\forall p \in P$ and $\forall\gamma \in \Gamma$, $\widehat{\alpha}(\gamma, p) > 0$ implies

$$1 \in \arg\max_{x\in\{0,1\}} x\sum_{\theta\in\Theta}[\gamma(\delta)\delta(\theta)\theta + \gamma(\widehat{\mu})\widehat{\mu}(\theta|p)\theta - p]$$

    and $\widehat{\alpha}(\gamma, p) < 1$ implies

$$0 \in \arg\max_{x\in\{0,1\}} x\sum_{\theta\in\Theta}[\gamma(\delta)\delta(\theta)\theta + \gamma(\widehat{\mu})\widehat{\mu}(\theta|p)\theta - p]$$

  - $\forall p \in P$,

$$\widehat{\pi}_p \in \arg\max_{\pi\in\Pi(\chi)} \sum_{\gamma\in\Gamma(\pi)} [\pi(\gamma)\widehat{\alpha}(\gamma, p)\sum_{\theta\in\Theta}[\gamma(\delta)\delta(\theta)\theta + \gamma(\widehat{\mu})\widehat{\mu}(\theta|p)\theta - p]] - K(\pi, \lambda, \chi)$$

- Bayesian beliefs on path (when not cursed)
  - If $\widehat{\sigma}(p|\theta) > 0$ for some $p \in P$, then

$$\widehat{\mu}(\theta|p) = \frac{\delta(\theta)\widehat{\sigma}(p|\theta)}{\sum_{\hat{\theta}\in\Theta}[\delta(\hat{\theta})\widehat{\sigma}(p|\hat{\theta})]}$$

6.1. Mimicking equilibrium

As with the case of a strategically sophisticated buyer who is inattentive to exogenous information about quality, there is a "mimicking" RICE in which the high quality seller always charges a high price of p_H, and the low quality seller mimics the high quality seller by charging this price with a certain probability and otherwise charges the low price p_L.

In fact, without cursedness there always exists such an equilibrium because if buyers are not willing to purchase when there is full pooling at a high price, then low quality sellers can always mimic at a rate which makes buyers indifferent. However, once cursedness is introduced, low quality sellers cannot always lower their mimicking rate enough to overcome this cursedness, which can prevent such an equilibrium from existing. On the other hand, with rational cursedness, the buyer can put in some attentional effort to reduce their strategic naïveté, which helps to reduce the power of cursedness to prevent this equilibrium from existing.

In this mimicking RICE, the attention strategy of the buyer makes both sellers indifferent between pricing high or low, and the rate at which a low quality seller prices high makes this attention strategy optimal. The following theorem establishes that if cursedness is sufficiently low relative to the parameters of the game, then there always exists such an equilibrium and indicates that for a given game G and cursedness χ, the rate of mimicking is uniquely pinned down.

Theorem 2. *For game G and cursedness χ, there exists a RICE in which high quality sellers set price p_H with probability 1 and low quality sellers set price p_H with a unique probability η and otherwise set price p_L if and only if*

$$\chi < \frac{\exp\left(\frac{\widehat{\mu}(\theta_H|p_H)\theta_H+(1-\widehat{\mu}(\theta_H|p_H))\theta_L-p_H}{\lambda}\right)-1}{\exp\left(\frac{\widehat{\mu}(\theta_H|p_H)\theta_H+(1-\widehat{\mu}(\theta_H|p_H))\theta_L-p_H}{\lambda}\right)-\exp\left(\frac{\delta\theta_H+(1-\delta)\theta_L-p_H}{\lambda}\right)}$$

where $\widehat{\mu}(\theta_H|p_H)$ *is defined according to:*

- *Seller pricing strategy*

$$\widehat{\sigma}(p_H|\theta_H)=1, \widehat{\sigma}(p_H|\theta_L)=\eta, \widehat{\sigma}(p_L|\theta_L)=1-\eta$$

- *Buyer attention strategy*

$$\widehat{\pi}_{p_H}(\gamma^1)=\min\left\{\frac{(1-\chi)-\gamma^0(\widehat{\mu})}{\gamma^1(\widehat{\mu})-\gamma^0(\widehat{\mu})}, 1\right\}, \widehat{\pi}_{p_H}(\gamma^0)=1-\widehat{\pi}_{p_H}(\gamma^1), \widehat{\pi}_{p_L}(\gamma^*)=1$$

- *Buyer purchasing strategy*

$$\widehat{\alpha}(\gamma^1, p_H)=1, \widehat{\alpha}(\gamma^0, p_H)=0, \widehat{\alpha}(\gamma^*, p_L)=1$$

- *Buyer beliefs (when not cursed)*

$$\widehat{\mu}(\theta_H|p_H)=\frac{\delta(\theta_H)}{\delta(\theta_H)+(1-\delta(\theta_H))\eta}, \widehat{\mu}(\theta_H|p_L)=0$$

- *Optimal posteriors*

$$\gamma^0(\widehat{\mu})=\min\left\{\frac{1-\exp\left(\frac{\delta\theta_H+(1-\delta)\theta_L-p_H}{\lambda}\right)}{\exp\left(\frac{\widehat{\mu}(\theta_H|p_H)\theta_H+(1-\widehat{\mu}(\theta_H|p_H))\theta_L-p_H}{\lambda}\right)-\exp\left(\frac{\delta\theta_H+(1-\delta)\theta_L-p_H}{\lambda}\right)}, 1-\chi\right\},$$

$$\gamma^1(\widehat{\mu})=\max\left\{\exp\left(\frac{\widehat{\mu}(\theta_H|p_H)\theta_H+(1-\widehat{\mu}(\theta_H|p_H))\theta_L-p_H}{\lambda}\right)\gamma^0(\widehat{\mu}), 1-\chi\right\}, \gamma^*(\widehat{\mu})=1-\chi$$

- *Mimicking rate*

$$\eta=\min\left\{\frac{\delta(\theta_H)}{1-\delta(\theta_H)}\frac{\theta_H-p_H-\lambda ln\left(\frac{(1-\chi)}{(1-\chi)-\frac{p_H}{p_L}((1-\chi)-\gamma^0(\widehat{\mu}))}\right)}{p_H-\theta_L+\lambda ln\left(\frac{(1-\chi)}{(1-\chi)-\frac{p_H}{p_L}((1-\chi)-\gamma^0(\widehat{\mu}))}\right)}, 1\right\}$$

Proof of this theorem is in Appendix A.

In this equilibrium, the buyer has no reason at a low price to attend to the informational content of prices because he will accept the offer for sure at that price, no matter what his belief might be of the seller's type. As a consequence, he remains χ-cursed.

What happens at a high price depends on model parameter values. When the parameter values are such that a buyer always accepts offers when both types of sellers pool at a high price, then the buyer never pays attention to the state μ (staying χ-cursed) and both types of sellers pool at the high price.[13]

The more interesting case is when a buyer would reject offers when both sellers pool at a high price. In this case, if buyers are not overly-cursed, then they will pay attention at a high price because at the Bayesian correct posterior, the buyer will purchase, but at the unconditional probability, the buyer will not purchase. The mimicking rate is key to ensuring that the buyer will purchase at the Bayesian correct posterior.

However, if cursedness is high enough, then no matter what sellers do, buyers will choose to remain χ-cursed and will not buy at a high price. This means that a mimicking equilibrium cannot exist. The condition given in the statement of Theorem 2 provides specifically how much cursedness is needed for this to occur.

There are very strong similarities between the mimicking equilibria for both types of inattention to quality. Not only are pricing strategies quite similar, but so are attentional strategies. When prices are high, the buyer's optimal information structure contains just two posteriors. One very notable difference is that for the mimicking RICE, the seller's strategy influences the buyer's optimal posteriors. As a consequence, the mimicking rate is quite different.

[13] This is true no matter the level of cursedness.

7. Concluding remarks

In this paper, I modify a simple and standard strategic pricing game by making buyers rationally inattentive to quality. As a result, attentional costs are a key driver in the pricing strategies of sellers, and hence in the informativeness of prices.

As an extension of this game, it would be interesting to consider what happens if the seller can influence the buyer's attentional costs (as in Carlin and Manso, 2011; Kalaycı and Potters, 2011; Perez-Richet and Prady, 2011) or can bias the information that is available to the buyer. Both possibilities seem realistic given the control that sellers often have over the retail environment. A complication of adding these features to the model is that the seller can communicate information through these actions, giving another channel over which the buyer must have beliefs, which can increase the set of equilibria. One solution employed in the literature is to have buyers be nonstrategic over these actions. Another possibility is to have sellers take these actions before they become aware of the quality of their product.

Appendix A

Proof of Theorem 1. The proof has four parts: 1) take an arbitrary $p_H \in (\theta_L, \theta_H)$ and find the optimal posteriors, 2) use these posteriors to find the conditional demands, 3) use low quality demands to find the unique mimicking rate, and 4) show that no agent has a profitable deviation.

1) This step involves a straightforward application of the ILR conditions from Caplin and Dean (2013). Let γ^0 be the posterior for which the buyer does not purchase and γ^1 be the posterior for which the buyer purchases. If I assume that both actions (purchase and not purchase) are taken with positive probability, the ILR conditions are

$$\frac{\gamma^1(\theta_H)}{\gamma^0(\theta_H)} = \frac{\exp(\frac{\theta_H - p_H}{\lambda})}{\exp(\frac{0}{\lambda})}$$

$$\frac{\gamma^1(\theta_L)}{\gamma^0(\theta_L)} = \frac{\exp(\frac{\theta_L - p_H}{\lambda})}{\exp(\frac{0}{\lambda})}$$

So the optimal posteriors for θ_H are

$$\gamma^0(\theta_H) = \frac{1 - \exp(\frac{\theta_L - p_H}{\lambda})}{\exp(\frac{\theta_H - p_H}{\lambda}) - \exp(\frac{\theta_L - p_H}{\lambda})}$$

$$\gamma^1(\theta_H) = \exp(\frac{\theta_H - p_H}{\lambda})\gamma^0(\theta_H)$$

2) Using Bayes' rule and the information structure that corresponds to these posteriors, it can be determined that conditional demands are

$$d(\theta_H|p_H) = \Pr(buy|\theta_H) = \frac{\gamma^1(\theta_H)\pi(\gamma^1)}{\mu(\theta_H|p_H)}$$

and

$$d(\theta_L|p_H) = \Pr(buy|\theta_L) = \frac{(1 - \gamma^1(\theta_H))\pi(\gamma^1)}{1 - \mu(\theta_H|p_H)}$$

The unconditional likelihood $\pi(\gamma^1)$ of posterior γ^1 is determined by strategic beliefs μ because

$$\pi(\gamma^1) = \frac{\mu(\theta_H|p_H) - \gamma^0(\theta_H)}{\gamma^1(\theta_H) - \gamma^0(\theta_H)}.$$

Note that for the assumption that both actions are taken to hold, it must be that $\gamma^0(\theta_H) < \mu(\theta_H|p_H) < \gamma^1(\theta_H)$. Given that attention is costly and $\gamma^1(\theta_H) \neq \gamma^0(\theta_H)$, it must be that the agent strictly prefers one of the actions after observing both posteriors.

If $\mu(\theta_H|p_H) \geq \gamma^1(\theta_H)$, which can occur if there is full pooling and a small number of low quality sellers, then the buyer optimally learns nothing, and so the assumption that both actions are taken is violated. In this case, $\pi(\gamma^1) = 1$ and $\gamma^1(\theta_H) = \delta(\theta_H)$. Also $\mu(\theta_H|p_H)$ cannot be less than or equal to $\gamma^0(\theta_H)$ in equilibrium, since then the buyer optimally learns nothing and never purchases at the high price, so sellers would have an incentive to deviate and charge a low price instead.

3) To find the mimicking rate η when there is not full pooling, we start with the indifference condition (of pricing low or high) for low quality sellers

$$p_H * d(\theta_L|p_H) = p_L$$

or

$$d(\theta_L|p_H) = \frac{(1-\gamma^1(\theta_H))\frac{\mu(\theta_H|p_H)-\gamma^0(\theta_H)}{\gamma^1(\theta_H)-\gamma^0(\theta_H)}}{(1-\mu(\theta_H|p_H))} = \frac{p_L}{p_H}$$

Rearranging this equation gives

$$\mu(\theta_H|p_H) = \frac{\frac{(1-\gamma^1(\theta_H))}{(\gamma^1(\theta_H)-\gamma^0(\theta_H))}\gamma^0(\theta_H) + \frac{p_L}{p_H}}{\frac{(1-\gamma^1(\theta_H))}{(\gamma^1(\theta_H)-\gamma^0(\theta_H))} + \frac{p_L}{p_H}}$$

To find η, we can replace the strategic probability that a seller is of high quality at the high price with

$$\mu(\theta_H|p_H) = \frac{\delta(\theta_H)}{\delta + (1-\delta(\theta_H))\eta}$$

To show that no other value of η supports the equilibrium, it is enough to show the existence of a single crossing property for $\frac{p_L}{p_H}$ and $d(\theta_L|p_H)$ as $\mu(\theta_H|p_H)$ increases. For some $\mu(\theta_H|p_H)$ less than $\delta(\theta_H)$ that is close to $\gamma^0(\theta_H)$, $d(\theta_L|p_H) < \frac{p_L}{p_H}$, and for some $\mu(\theta_H|p_H) > \delta(\theta_H)$ that is close to $\gamma^1(\theta_H)$, $d(\theta_L|p_H) > \frac{p_L}{p_H}$. Thus, because $d(\theta_L|p_H)$ is strictly increasing in $\mu(\theta_H|p_H)$, there exists a single $\mu(\theta_H|p_H)$ where $d(\theta_L|p_H) = \frac{p_L}{p_H}$.

To show that both actions are taken for $\eta < 1$, note that

$$\mu(\theta_H|p_H) = \frac{\frac{(1-\gamma^1(\theta_H))}{(\gamma^1(\theta_H)-\gamma^0(\theta_H))}\gamma^0(\theta_H) + \frac{p_L}{p_H}}{\frac{(1-\gamma^1(\theta_H))}{(\gamma^1(\theta_H)-\gamma^0(\theta_H))} + \frac{p_L}{p_H}} > \gamma^0(\theta_H)$$

and

$$\mu(\theta_H|p_H) = \frac{\frac{(1-\gamma^1(\theta_H))}{(\gamma^1(\theta_H)-\gamma^0(\theta_H))}\gamma^0(\theta_H) + \frac{p_L}{p_H}}{\frac{(1-\gamma^1(\theta_H))}{(\gamma^1(\theta_H)-\gamma^0(\theta_H))} + \frac{p_L}{p_H}} < \gamma^1(\theta_H)$$

because $\frac{p_L}{p_H} > \frac{p_L}{p_H}\gamma^0(\theta_H)$ and $(\gamma^1(\theta_H)-1) < \frac{p_L}{p_H}(\gamma^1(\theta_H)-1)$.

4) It is optimal to pay no attention and buy the good at a price of p_L because buyers do weakly better by purchasing at this price. By indifference, low quality sellers get an expected return of p_L no matter what price they set, so they have no incentive to deviate. Because their conditional demand is weakly higher, higher quality sellers get a higher return, so will not deviate to pricing low. □

Proof of Proposition 1. This proof has two parts. First, I show that the uncapped rate of mimicking converges to $\frac{\delta(\theta_H)}{1-\delta(\theta_H)}\frac{\theta_H-p_H}{p_H-\theta_L}$ in the limit as the marginal cost of attention λ goes to infinity. Thus, if the uncapped rate of mimicking converges above one (if $\delta(\theta_H) > \frac{p_H-\theta_L}{\theta_H-\theta_L}$), then the capped rate of mimicking will converge to one, and if the uncapped rate of mimicking converges below one (if $\delta(\theta_H) < \frac{p_H-\theta_L}{\theta_H-\theta_L}$), then the capped rate will as well. Second, I show that the uncapped rate of mimicking is decreasing in the limit as λ goes to infinity if (and only if) $p_H > 2p_L$.

1) From the proof of Proposition 1, the uncapped rate of mimicking can be written as

$$\frac{\delta(\theta_H)}{1-\delta(\theta_H)}\frac{\exp(\frac{\theta_H}{\lambda}) - \exp(\frac{p_H}{\lambda})}{\exp(\frac{p_H}{\lambda}) - \exp(\frac{\theta_L}{\lambda})}\frac{1}{1 + \frac{p_L}{p_H}\left(\exp(\frac{\theta_H-\theta L}{\lambda}) - 1\right)}$$

First, note that $\exp(\frac{\theta_H-\theta_L}{\lambda}) - 1$ converges to 0 in the limit as λ goes to infinity.
Second, note that by L'Hospital's Rule,

$$\lim_{\lambda\to\infty}\frac{\exp(\frac{\theta_H}{\lambda}) - \exp(\frac{p_H}{\lambda})}{\exp(\frac{p_H}{\lambda}) - \exp(\frac{\theta_L}{\lambda})} = \lim_{\lambda\to\infty}\frac{\theta_H\exp(\frac{\theta_H}{\lambda}) - p_H\exp(\frac{p_H}{\lambda})}{p_H\exp(\frac{p_H}{\lambda}) - \theta_L\exp(\frac{\theta_L}{\lambda})} = \frac{\theta_H - p_H}{p_H - \theta_L}$$

Thus, the uncapped rate of mimicking converges in the limit, as λ goes to infinity, to

$$\frac{\delta(\theta_H)}{1-\delta(\theta_H)}\frac{\theta_H - p_H}{p_H - \theta_L}$$

2) The derivative in terms of λ for the uncapped rate of mimicking is

$$\frac{\delta(\theta_H)}{1-\delta(\theta_H)}\frac{1}{\lambda^2\left(1+\frac{p_L}{p_H}\left(\exp(\frac{\theta_H-\theta L}{\lambda})-1\right)\right)}*$$
$$\left[\frac{(\exp(\frac{\theta_H}{\lambda})-\exp(\frac{p_H}{\lambda}))(p_H\exp(\frac{p_H}{\lambda})-\theta_L\exp(\frac{\theta_L}{\lambda}))-(\theta_H\exp(\frac{\theta_H}{\lambda})-p_H\exp(\frac{p_H}{\lambda}))(\exp(\frac{p_H}{\lambda})-\exp(\frac{\theta_L}{\lambda}))}{(\exp(\frac{p_H}{\lambda})-\exp(\frac{\theta_L}{\lambda}))^2}\right.$$
$$\left.+\frac{\exp(\frac{\theta_H}{\lambda})-\exp(\frac{p_H}{\lambda})}{\exp(\frac{p_H}{\lambda})-\exp(\frac{\theta_L}{\lambda})}\frac{\frac{p_L}{p_H}(\theta_H-\theta_L)\exp(\frac{\theta_H-\theta_L}{\lambda})}{1+\frac{p_L}{p_H}\left(\exp(\frac{\theta_H-\theta L}{\lambda})-1\right)}\right]$$

Applying L'Hospital's Rule again, the first term inside of the brackets of the derivative converges in the limit, as λ goes to infinity, to

$$-\frac{1}{2}\frac{\theta_H-p_H}{p_H-\theta_L}(\theta_H-\theta_L)$$

and the second term inside of the brackets of the derivative converges to

$$\frac{p_L}{p_H}\frac{\theta_H-p_H}{p_H-\theta_L}(\theta_H-\theta_L)$$

Thus, the expression inside of the brackets becomes negative as it converges in the limit if (and only if) $p_H > 2p_L$. The expression outside of the brackets converges to 0 in the limit as λ goes to infinity, so the derivative converges to 0 from below if (and only if) $p_H > 2p_L$. □

Proof of Theorem 2. The proof has four parts: 1) take an arbitrary $p_H \in (\theta_L, \theta_H)$ and find the optimal posteriors, 2) use these posteriors to find the demand given the true state, 3) use this demand to find the unique mimicking rate, and 4) show that no agent has a profitable deviation.

1) This step involves a straightforward application of the ILR conditions from Caplin and Dean (2013). Let γ^0 be the posterior for which the buyer does not purchase and γ^1 be the posterior for which the buyer purchases. If I assume that both actions (purchase and not purchase) are taken with positive probability, the ILR conditions are

$$\frac{\gamma^1(\widehat{\mu})}{\gamma^0(\widehat{\mu})}=\frac{\exp\left(\frac{\widehat{\mu}(\theta_H|p_H)\theta_H+(1-\widehat{\mu}(\theta_H|p_H))\theta_L-p_H}{\lambda}\right)}{\exp\left(\frac{0}{\lambda}\right)}=A$$
$$\frac{\gamma^1(\delta)}{\gamma^0(\delta)}=\frac{\exp\left(\frac{\delta\theta_H+(1-\delta)\theta_L-p_H}{\lambda}\right)}{\exp\left(\frac{0}{\lambda}\right)}=B$$

So the optimal posteriors for $\widehat{\mu}$ are

$$\gamma^0(\widehat{\mu})=\frac{1-\exp\left(\frac{\delta\theta_H+(1-\delta)\theta_L-p_H}{\lambda}\right)}{\exp\left(\frac{\widehat{\mu}(\theta_H|p_H)\theta_H+(1-\widehat{\mu}(\theta_H|p_H))\theta_L-p_H}{\lambda}\right)-\exp\left(\frac{\delta\theta_H+(1-\delta)\theta_L-p_H}{\lambda}\right)}=\frac{1-B}{A-B}$$
$$\gamma^1(\widehat{\mu})=\exp\left(\frac{\widehat{\mu}(\theta_H|p_H)\theta_H+(1-\widehat{\mu}(\theta_H|p_H))\theta_L-p_H}{\lambda}\right)\gamma^0(\widehat{\mu})=\frac{A(1-B)}{A-B}$$

2) Using Bayes' rule and the information structure that corresponds to these posteriors, it can be determined that the unconditional likelihood $\pi(\gamma^1)$ of posterior γ^1 (if the buyer's beliefs were correct) is determined by χ because

$$\pi(\gamma^1)=\frac{(1-\chi)-\gamma^0(\widehat{\mu})}{\gamma^1(\widehat{\mu})-\gamma^0(\widehat{\mu})}.$$

Note that for the assumption that both actions are taken to hold, it must be that $\gamma^0(\widehat{\mu}) < 1-\chi < \gamma^1(\widehat{\mu})$. Given that attention is costly and $\gamma^1(\widehat{\mu}) \neq \gamma^0(\widehat{\mu})$, it must be that the agent strictly prefers one of the actions after observing both posteriors. If $1-\chi \geq \gamma^1(\widehat{\mu})$, which can occur if there is a little cursedness, then the buyer optimally learns nothing, and so the assumption that both actions are taken is violated. In this case, $\pi(\gamma^1)=1$. Also $1-\chi$ cannot be less than or equal to $\gamma^0(\widehat{\mu})$ in equilibrium, since then the buyer optimally learns nothing and never purchases at the high price, so sellers would have an incentive to deviate and charge a low price instead.

Because $\widehat{\mu}$ is the true state, the demand for the seller's product at a high price is determined by

$$\Pr(buy|\widehat{\mu}) = \pi(\gamma^1|\widehat{\mu})$$

By Bayes' rule, demand is related to the posteriors by the following equations

$$\gamma^1(\widehat{\mu}) = \frac{\pi(\gamma^1|\widehat{\mu})(1-\chi)}{\pi(\gamma^1|\widehat{\mu})(1-\chi)+\pi(\gamma^1|\delta)\chi}$$

$$\gamma^0(\widehat{\mu}) = \frac{(1-\pi(\gamma^1|\widehat{\mu}))(1-\chi)}{(1-\pi(\gamma^1|\widehat{\mu}))(1-\chi)+(1-\pi(\gamma^1|\delta))\chi}$$

From these equations, it can be determined that

$$\pi(\gamma^1|\widehat{\mu}) = \frac{(1-\chi)-\gamma^0(\widehat{\mu})}{(1-\chi)\left(1-\frac{1}{\exp\left(\frac{\widehat{\mu}(\theta_H|p_H)\theta_H+(1-\widehat{\mu}(\theta_H|p_H))\theta_L-p_H}{\lambda}\right)}\right)}$$

or

$$\pi(\gamma^1|\widehat{\mu}) = \frac{(1-\chi)-\frac{1-B}{A-B}}{(1-\chi)\left(1-\frac{1}{A}\right)} = \frac{1}{(1-\chi)}\frac{(1-\chi)A-(1-\chi B)}{A-1-B+\frac{B}{A}}$$

Note that $\pi(\gamma^1|\widehat{\mu})$ is strictly increasing in $\widehat{\mu}(\theta_H|p_H)$ because the derivative with respect to $\widehat{\mu}(\theta_H|p_H)$ is always positive. This derivative is positive if

$$\chi > \frac{B(A-1)^2}{(A-B)^2}$$

When attention is paid, $1-\chi < \gamma^1$ or $\chi > 1-\gamma^1$, so

$$\chi > \frac{B(A-1)}{(A-B)} = \frac{B(A-1)(A-B)}{(A-B)^2}$$

which is larger than the desired condition because $B = \exp\left(\frac{\delta\theta_H+(1-\delta)\theta_L-p_H}{\lambda}\right) < 1$.

3) To find the mimicking rate η when there is not full pooling, we start with the indifference condition (of pricing low or high) for all sellers (both high and low quality)

$$p_H * \Pr(buy|\widehat{\mu}) = p_L$$

or

$$\frac{p_L}{p_H} = \Pr(buy|\widehat{\mu}) = \frac{(1-\chi)-\gamma^0(\widehat{\mu})}{(1-\chi)\left(1-\frac{1}{\exp\left(\frac{\widehat{\mu}(\theta_H|p_H)\theta_H+(1-\widehat{\mu}(\theta_H|p_H))\theta_L-p_H}{\lambda}\right)}\right)}$$

Rearranging this equation gives

$$\exp\left(\frac{\widehat{\mu}(\theta_H|p_H)\theta_H+(1-\widehat{\mu}(\theta_H|p_H))\theta_L-p_H}{\lambda}\right) = \left(\frac{(1-\chi)}{(1-\chi)-\frac{p_H}{p_L}((1-\chi)-\gamma^0(\widehat{\mu}))}\right) = C$$

or

$$\widehat{\mu}(\theta_H|p_H) = \frac{p_H-\theta_L+\lambda\ln(C)}{\theta_H-\theta_L}$$

To find η, we can replace the strategic probability that a seller is of high quality at the high price with

$$\widehat{\mu}(\theta_H|p_H) = \frac{\delta(\theta_H)}{\delta+(1-\delta(\theta_H))\eta}$$

Which gives

$$\eta = \frac{\delta(\theta_H)}{1-\delta(\theta_H)}\frac{\theta_H-p_H-\lambda ln\left(\frac{(1-\chi)}{(1-\chi)-\frac{p_H}{p_L}((1-\chi)-\gamma^0(\widehat{\mu}))}\right)}{p_H-\theta_L+\lambda ln\left(\frac{(1-\chi)}{(1-\chi)-\frac{p_H}{p_L}((1-\chi)-\gamma^0(\widehat{\mu}))}\right)}$$

Because $\Pr(buy|\widehat{\mu})$ is strictly increasing in $\widehat{\mu}$, there is a single crossing property for $\frac{p_L}{p_H}$ and $\Pr(buy|\widehat{\mu})$ as $\widehat{\mu}$ increases (for values of χ that satisfy the condition given).

To show that both actions are taken for $\eta < 1$, note that

$$\gamma^1(\widehat{\mu}) = \frac{\pi(\gamma^1|\widehat{\mu})(1-\chi)}{\pi(\gamma^1|\widehat{\mu})(1-\chi)+\pi(\gamma^1|\delta)\chi} > 1-\chi$$

$$\gamma^0(\widehat{\mu}) = \frac{(1-\pi(\gamma^1|\widehat{\mu}))(1-\chi)}{(1-\pi(\gamma^1|\widehat{\mu}))(1-\chi)+(1-\pi(\gamma^1|\delta))\chi} < 1-\chi$$

because $\pi(\gamma^1|\widehat{\mu}) > \pi(\gamma^1|\delta)$.

4) It is optimal to pay no attention and buy the good at a price of p_L because buyers do weakly better by purchasing at this price. By indifference, sellers get an expected return of p_L no matter what price they set, so they have no incentive to deviate. □

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